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Mathematical Sciences

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PART A

Section Id :	128206165
Section Number :	1
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Mandatory or Optional:	Mandatory
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Question Shuffling Allowed :	Yes

Question Number : 1 Question Id : 1282065984 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let (X, d_X) and (Y, d_Y) be metric spaces, $f : X \to Y$ be a map and $x_0 \in X$. For $\delta > 0$, let $B_\delta(x_0) := \{x \in X : d_X(x, x_0) < \delta\}$. Which one of the following statements is the correct negation of continuity of f at x_0?

(a) For every $\epsilon > 0$, there exists a $\delta > 0$ such that $d_Y(f(x), f(x_0)) > \epsilon$ for all $x \in B_\delta(x_0)$.

(b) There exists an $\epsilon > 0$ and a $\delta > 0$ such that $d_Y(f(x), f(x_0)) > \epsilon$ for all $x \in B_\delta(x_0)$.

(c) There exists an $\epsilon > 0$ such that for every $\delta > 0$ there exists an $x \in B_\delta(x_0)$ satisfying $d_Y(f(x), f(x_0)) > \epsilon$.

(d) There exists an $\epsilon > 0$ such that for every $\delta > 0$ we have $d_Y(f(x), f(x_0)) > \epsilon$ for all $x \in B_\delta(x_0)$.

Options :

12820623675. A

12820623676. B

12820623677. C

12820623678. D

Question Number : 2 Question Id : 1282065985 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Which one of the following relations is an equivalence relation?

(a) For $x, y \in \mathbb{R}$, say that $x \sim y$ if $x - y$ is a rational number.

(b) For $x, y \in \mathbb{R}$, say that $x \sim y$ if $x - y$ is an irrational number.

(c) Let X be the set of human beings on Earth. For $A, B \in X$, say that $A \sim B$ if A is a brother of B.

(d) None of the above relations is an equivalence relation.

Options :

12820623679. A

12820623680. B

12820623681. C

12820623682. D

Question Number : 3 Question Id : 1282065986 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Which one of the following statements is necessarily true?

(a) Arbitrary intersection of uncountable subsets of a set is uncountable.

(b) Arbitrary intersection of connected subsets of a topological space is connected.

(c) Arbitrary union of closed subsets of a topological space is closed.

(d) None of the above statements is true.

Options :

12820623683. A

12820623684. B

12820623685. C

12820623686. D

Question Number : 4 Question Id : 1282065987 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 3 Wrong Marks : 0

Let $n \in \mathbb{N}$ and $\mathcal{F} := \{A \in M_n(\mathbb{C}) : A^5 = I\}$. Then,

(a) $|\mathcal{F}| = 5$.

(b) $|\mathcal{F}| = 5n$.

(c) $|\mathcal{F}| = n^5$.

(d) $\mathcal{F}$ is infinite.

Options :

12820623687. A

12820623688. B

12820623689. C

12820623690. D

Question Number : 5 Question Id : 1282065988 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 3 Wrong Marks : 0

Let W_1 and W_2 be subspaces of $\mathbb{R}^4$ given by

$$W_1 = \{(x, y, z, w) : x = -z\} \text{ and } W_2 = \{(x, y, z, w) : y = z = 0, w = x\}.$$

Then, $\dim(W_1 + W_2)$ equals

(a) 4

(b) 3

(c) 2

(d) 1

Options :

12820623691. A

12820623692. B

12820623693. C

12820623694. D

Question Number : 6 Question Id : 1282065989 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 3 Wrong Marks : 0

Let $V = \mathbb{C}^n$ with $n \geq 3$. If v is a unit vector in V, then how many orthonormal bases of V contain v?

(a) 1

(b) n

(c) 2^{n-1}

(d) infinitely many

Options :

12820623695. A

12820623696. B

12820623697. C

12820623698. D

Question Number : 7 Question Id : 1282065990 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let S_n denote the permutation group on n symbols. Which one of the following assertions is true?

(a) S_{11} contains no element of order 12.

(b) If $\sigma \in S_5$ has order 5, then σ generates S_5.

(c) S_7 contains an element of order 12.

(d) If $\tau \in S_{18}$ has order 18, then τ generates S_{18}.

Options :

12820623699. A

12820623700. B

12820623701. C

12820623702. D

Question Number : 8 Question Id : 1282065991 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Upto isomorphism, how many commutative groups of order 8 are there?

(a) 1

(b) 2

(c) 3

(d) None of the above

Options :

12820623703. A

12820623704. B

12820623705. C

12820623706. D

Question Number : 9 Question Id : 1282065992 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Which one of the following assertions is **false**?

(a) For $a, b \in \mathbb{C}$, consider the map $\psi_{a,b} : \mathbb{C}[x, y] \to \mathbb{C}$ given by $\psi_{a,b}(f(x, y)) = f(a, b)$. Then, $\ker(\psi_{a,b})$ is a maximal ideal in $\mathbb{C}[x, y]$.

(b) For $a \in \mathbb{Z}$, consider the map $\psi_a : \mathbb{Z}[x, y] \to (\mathbb{Z}/5\mathbb{Z})[y]$ given by

$$\psi_a(f(x, y)) = f(a, y) \mod 5.$$

Then, $\ker(\psi_a)$ is a prime ideal in $\mathbb{Z}[x, y]$ which is not a maximal ideal.

(c) For $b \in \mathbb{R}$, consider the map $\psi_b : \mathbb{R}[x, y] \to \mathbb{R}[x]$ given by $\psi_b(f(x, y)) = f(x, b)$. Then, $\ker(\psi_b)$ is a prime ideal in $\mathbb{R}[x, y]$.

(d) $I := \{f \in \mathbb{C}[x] : f(0) = 0 \text{ and } f'(0) = 0\}$ is a maximal ideal in $\mathbb{C}[x]$.

Options :

12820623707. A

12820623708. B

12820623709. C

12820623710. D

Question Number : 10 Question Id : 1282065993 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let X be a metric space which is connected and let $f : X \to \mathbb{R}$ be any map. Which one of the following statements is necessarily true?

(a) If $f(X) \subseteq \mathbb{Q}$, then f is not continuous.

(b) If f is continuous and $f(X)$ is a finite set, then f is a constant map.

(c) If $f(X)$ is compact, then f is continuous.

(d) If f is injective and continuous, then X cannot be compact.

Options :

12820623711. A

12820623712. B

12820623713. C

12820623714. D

Question Number : 11 Question Id : 1282065994 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

For each $n \in \mathbb{N}$, consider the map $f_n : [-1, 1] \to \mathbb{R}$ given by

$$f_n(x) = \frac{5x^{4n}}{\pi + 5x^{4n}}.$$

Which one of the following assertions is true?

(a) $\{f_n\}$ converges pointwise to a continuous functions on $[-1, 1]$.

(b) $\{f_n\}$ converges uniformly to a continuous functions on $[-1, 1]$.

(c) $\{f_n\}$ converges pointwise to a non-continuous functions on $[-1, 1]$.

(d) None of the above is true.

Options :

12820623715. A

12820623716. B

12820623717. C

12820623718. D

Question Number : 12 Question Id : 1282065995 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let $f : \mathbb{C} \to \mathbb{C}$ be the map given by $f(z) = z^3\bar{z}$. Then,

(a) f is complex differentiable only at 0.

(b) f is complex differentiable everywhere.

(c) f is nowhere complex differentiable everywhere.

(d) f is complex differentiable on $\mathbb{R}$ only.

Options :

12820623719. A

12820623720. B

12820623721. C

12820623722. D

Question Number : 13 Question Id : 1282065996 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let $f, g : \mathbb{C} \to \mathbb{C}$ be two analytic functions satisfying

$$f(n) = n^2 \text{ and } g\left(\frac{1}{n}\right) = \frac{1}{n^2} \text{ for all } n \in \mathbb{N}. \qquad \text{Then}$$

(a) $f(z) = g(z) = z^2$ for all $z \in \mathbb{C}$.

(b) $f(z) = z^2$ for all $z \in \mathbb{C}$ but $g(z)$ cannot be determined uniquely.

(c) $g(z) = z^2$ for all $z \in \mathbb{C}$ but $f(z)$ cannot be determined uniquely.

(d) neither $f(z)$ nor $g(z)$ can be determined uniquely.

Options :

12820623723. A

12820623724. B

12820623725. C

12820623726. D

Question Number : 14 Question Id : 1282065997 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 3 Wrong Marks : 0

Let X be a complex normed space. Consider X as a topological space with the topology induced by its norm. Which one of the following statements is true?

(a) If X is finite dimensional, then every bounded subset of X is compact.

(b) If X is infinite dimensional, then every non-empty open subset of X has non-compact closure.

(c) If X is finite dimensional, then every closed subset of X is compact.

(d) If X is infinite dimensional, then the open unit ball of X is contained in a compact set.

Options :

12820623727. A

12820623728. B

12820623729. C

12820623730. D

Question Number : 15 Question Id : 1282065998 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 3 Wrong Marks : 0

What is the remainder when 3^{26} is divided by 35?

(a) 1

(b) 3

(c) 9

(d) None of the above

Options :

12820623731. A

12820623732. B

12820623733. C

12820623734. D

PART B

Section Id :	128206166
Section Number :	2
Section type :	Online
Mandatory or Optional:	Mandatory
Number of Questions:	11
Number of Questions to be attempted:	11
Section Marks:	55
Display Number Panel:	Yes
Group All Questions:	No

Sub-Section Number: 1
Sub-Section Id: 128206271
Question Shuffling Allowed : Yes

Question Number : 16 Question Id : 1282065999 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 5 Wrong Marks : 0

Let $a \in \mathbb{C}$. Then, the system of linear equations

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \\ x_1 - x_2 + x_3 - x_4 &= a \\ x_1 + 2x_2 + 3x_3 + 3x_4 &= 3 \\ 4x_1 + 3x_2 + 2x_3 + x_4 &= 4 \end{aligned}$$

has

(a) a unique solution for every $a \in \mathbb{C}$.
(b) infinitely many solutions for every $a \in \mathbb{C}$.
(c) no solution for any $a \in \mathbb{C}$.
(d) a unique solution for $a = 2$ and no solution for $a \neq 2$.

Options :
12820623735. A
12820623736. B
12820623737. C
12820623738. D

Question Number : 17 Question Id : 1282066000 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 5 Wrong Marks : 0

Let $A, B \in M_2(\mathbb{C})$ and $C := AB - BA$. Which one of the following statements is necessarily true?

(a) C is diagonalizable.
(b) Eigenvalues of C are all distinct.
(c) Minimal polynomial of C is the same as its characteristic polynomial.
(d) If C is not invertible, then it must be nilpotent, i.e., $C^k = 0$ for some $k \in \mathbb{N}$.

Options :
12820623739. A
12820623740. B
12820623741. C
12820623742. D

Question Number : 18 Question Id : 1282066001 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 5 Wrong Marks : 0

Let F be a field with 125 elements and G be the multiplicative group associated to F. Let k denote the number of elements in G of order $|G|$. Which one of the following assertions is true?

(a) $|G| = 124$ and $k = 0$.

(b) $|G| = 100$ and $k = 40$.

(c) $|G| = 124$ and $k = 60$.

(d) $|G| = 124$ and $k = 1$.

Options :

12820623743. A

12820623744. B

12820623745. C

12820623746. D

Question Number : 19 Question Id : 1282066002 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Consider the polynomials

$$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)-1,$$
$$g(x) = (x+1)^4+(x+1)^3+(x+1)^2+(x+1)+1 \text{ and}$$
$$h(x) = \frac{x^7-1}{x-1}$$

with coefficients in $\mathbb{Z}$. Which one of the following assertions is true?

(a) f and g are irreducible, and h is reducible over $\mathbb{Z}[x]$

(b) f, g and h are all irreducible over $\mathbb{Z}[x]$

(c) g and h are irreducible, and f is reducible over $\mathbb{Z}[x]$

(d) f, g and h are all reducible over $\mathbb{Z}[x]$

Options :

12820623747. A

12820623748. B

12820623749. C

12820623750. D

Question Number : 20 Question Id : 1282066003 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Consider $(\mathbb{R}^2, d)$ as a metric space where d is the usual metric on $\mathbb{R}^2$. Let $\mathcal{K}$ denote the set of compact subsets of $\mathbb{R}^2$. Which one of the following expressions gives a metric on $\mathcal{K}$?

(a) $d_1(K_1, K_2) := \inf_{x\in K_1, y\in K_2} d(x,y)$

(b) $d_2(K_1, K_2) := \inf_{x\in K_1} \sup_{y\in K_2} d(x,y)$

(c) $d_3(K_1, K_2) := \sup_{x\in K_1} \inf_{y\in K_2} d(x,y)$

(d) None of the above

Options :

12820623751. A

12820623752. B

12820623753. C

12820623754. D

Question Number : 21 Question Id : 1282066004 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by

$$f(x) = \begin{cases} (x+\pi)^{-2} & \text{if } x \in \mathbb{Q}, \text{ and} \\ x^{-2} & \text{if } \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Let $\alpha = \int_1^2 f(x)dx$. Which one of the following assertions is true?

(a) f is continuous only at 0 and $\alpha = \frac{1}{2}$.

(b) f is continuous only at 0 and $\alpha = \frac{1}{(1+\pi)(2+\pi)}$.

(c) f is not continuous anywhere on $\mathbb{R}$ and $\alpha = \frac{1}{2}$.

(d) f is not continuous anywhere on $\mathbb{R}$ and $\alpha = \frac{1}{(1+\pi)(2+\pi)}$.

Options :

12820623755. A

12820623756. B

12820623757. C

12820623758. D

Question Number : 22 Question Id : 1282066005 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let $\Omega = (0, \frac{\pi}{2}) \times (0, \frac{\pi}{2})$ and consider the map $f : \Omega \to \mathbb{R}^2$ given by

$$f((x, y)) = \left(\int_0^x \sin(t)\, dt, \int_0^y \cos(t)\, dt \right).$$

Which one of the following assertions is true?

(a) f is differentiable on Ω and the derivative of f at $(\pi/4, \pi/4)$ is not invertible.

(b) f is differentiable on Ω and the derivative of f at $(\pi/4, \pi/4)$ is invertible.

(c) f is continuous but not differentiable on Ω.

(d) None of the above assertions are true.

Options :

12820623759. A

12820623760. B

12820623761. C

12820623762. D

Question Number : 23 Question Id : 1282066006 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

What is the value of the integral

$$\int_{|z|=2} \frac{(z+\frac{\pi}{2})}{1-\cos(z+\frac{\pi}{2})} dz\,?$$

(a) 0

(b) 2

(c) $2\pi i$

(d) $4\pi i$

Options :

12820623763. A

12820623764. B

12820623765. C

12820623766. D

Question Number : 24 Question Id : 1282066007 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let X be an infinite dimensional complex normed space and Y be a proper non-trivial subspace of X. Which of the following statements is necessarily true?

(a) If Y is finite dimensional, then every linear map $T : Y \to \mathbb{C}$ extends to a continuous linear map $\tilde{T} : X \to \mathbb{C}$.

(b) Every non-zero linear map $T : X \to \mathbb{C}$ is continuous and an open map.

(c) Every non-zero linear map $T : X \to Y$ is continuous.

(d) Y is always a closed and non-compact subset of X.

Options :

12820623767. A

12820623768. B

12820623769. C

12820623770. D

Question Number : 25 Question Id : 1282066008 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let V be a 4-dimensional complex vector space and $\{v_1, v_2, v_3, v_4\}$ be a basis for V. Let $T : V \to V$ be a linear map given by

$$\begin{aligned} T(v_1) &= v_1, \\ T(v_2) &= 2v_2 + v_3, \\ T(v_3) &= v_3, \\ T(v_4) &= 2v_2 + 3v_3 + 5v_4. \end{aligned}$$

Then, the minimal polynomial of T is

(a) $(x^2 - 1)(x - 2)(x - 5)$.
(b) $(x - 1)(x - 2)(x - 5)$.
(c) $(x - 1)^2(x - 2)(x - 5)$.
(d) $(x^2 + 1)(x - 2)(x - 5)$.

Options :

12820623771. A

12820623772. B

12820623773. C

12820623774. D

Question Number : 26 Question Id : 1282066009 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

The set of all solutions to the congruences

$$\begin{aligned} x &\equiv 3 \mod 12 \quad \text{and} \\ x &\equiv 5 \mod 26 \end{aligned}$$

is

(a) $\{31 + 52\,n \ : \ n \in \mathbb{Z}\}$.
(b) $\{57 + 78\,n \ : \ n \in \mathbb{Z}\}$.
(c) $\{135 + 156\,n \ : \ n \in \mathbb{Z}\}$.
(d) $\{135 + 312\,n \ : \ n \in \mathbb{Z}\}$.

Options :

12820623775. A

12820623776. B

12820623777. C

12820623778. D