

National Testing Agency

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Mathematics 237

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PART 1

Section Id : 128206325
Section Number : 1
Section type : Online
Mandatory or Optional: Mandatory
Number of Questions: 11
Number of Questions to be attempted: 11
Section Marks: 55
Display Number Panel: Yes
Group All Questions: No

Sub-Section Number: 1
Sub-Section Id: 128206524
Question Shuffling Allowed : Yes

Question Number : 1 Question Id : 12820611350 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 5 Wrong Marks : 0

Let $B := \{(0, 1, 1), (-1, 1, 0), (1, 1, 0)\}$. Let T be a linear transformation from \mathbb{R}^3 to \mathbb{R}^3 given by $T(x, y, z) := (x + y + z, 2y + z, 3z)$.

Which one of the following is true?

(a) B is not a basis

(b) B is a basis for \mathbb{R}^3 and the matrix of T with respect to B is $\begin{pmatrix} 0 & 2 & 4 \\ -1 & 1 & 0 \\ 1 & 3 & 2 \end{pmatrix}$

(c) B is a basis for \mathbb{R}^3 and the matrix of T with respect to B is $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$

(d) B is a basis for \mathbb{R}^3 and the matrix of T with respect to B is $\begin{pmatrix} 3 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

Options :

12820644847. A

12820644848. B

12820644849. C

12820644850. D

Question Number : 2 Question Id : 12820611351 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let $f, g : [0, \infty) \rightarrow \mathbb{R}$ be continuous functions satisfying the following equations

$$\int_0^{x^2} f(t) dt = x^2(1+x), \quad \int_0^{x^2(1+x)} g(t) dt = x^2.$$

Which one of the following is necessarily true?

(a) $f(x)^2 g(x)^2 (1+g(x)) = 1$

(b) $f(x^2) g(x^2(1+x)) = 1$

(c) $f(x)^2 / (g(x)^2 (1+g(x))) = 1$

(d) $f(x^2) = g(x^2(1+x))$

Options :

12820644851. A

12820644852. B

12820644853. C

12820644854. D

Question Number : 3 Question Id : 12820611352 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

The series $\sum_{n=1}^{\infty} \log \cos\left(\frac{1}{n}\right)$

(a) converges absolutely

(b) converges conditionally but not absolutely

(c) diverges to $+\infty$

(d) diverges to $-\infty$

Options :

12820644855. A

12820644856. B
12820644857. C
12820644858. D

Question Number : 4 Question Id : 12820611353 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Consider the following statements:

- [S1] The number of groups of order 15 up to isomorphism is 2.
- [S2] The number of groups of order 15 up to isomorphism is 1.
- [S3] All groups of order 15 are Abelian.
- [S4] There is a group of order 15 which is non-Abelian.

Which one of the following is necessarily true?

- (a) Only [S1] and [S4]
- (b) Only [S1] and [S3]
- (c) Only [S2] and [S3]
- (d) Only [S3]

Options :

12820644859. A
12820644860. B
12820644861. C
12820644862. D

Question Number : 5 Question Id : 12820611354 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Consider the series S_1 and S_2 given by

$$S_1 := \sum_{n=1}^{\infty} \left(1 - \frac{1}{2^n}\right)^n$$
$$S_2 := \sum_{n=1}^{\infty} \frac{(-1)^n \log^3 n}{n}.$$

Which one of the following is true?

- (a) S_1 converges and S_2 converges absolutely
- (b) S_1 converges and S_2 converges conditionally but not absolutely
- (c) S_1 converges and S_2 diverges
- (d) S_1 diverges and S_2 converges

Options :

12820644863. A
12820644864. B
12820644865. C
12820644866. D

Question Number : 6 Question Id : 12820611355 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Consider the integral $I := \int_C \frac{-ydx + xdy}{x^2 + y^2}$ calculated counterclockwise around the circle C centred at $(0, 3)$ and radius 2. Then, the value of I equals

- (a) 0
- (b) 2π
- (c) -2π
- (d) None of the above

Options :

- 12820644867. A
- 12820644868. B
- 12820644869. C
- 12820644870. D

Question Number : 7 Question Id : 12820611356 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

For subsets T_1 and T_2 of \mathbb{R} , we define $T_1 + T_2 := \{t_1 + t_2 \mid t_1 \in T_1, t_2 \in T_2\}$. Let $S_1 := (\mathbb{Q} \cap [0, 1]) + \{\sqrt{2}\}$ and $S_2 := (\mathbb{Q} \cap [1, 2]) + \{\sqrt{3}\}$. Let $S = S_1 \cup S_2$.

Which one of the following is true?

- (a) There exists a one to one map from $[0, 1]$ to S
- (b) There exists an onto map from S to $[0, 1]$
- (c) $\#S_1 = \#S_2$
- (d) None of the above

Options :

- 12820644871. A
- 12820644872. B
- 12820644873. C
- 12820644874. D

Question Number : 8 Question Id : 12820611357 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Consider the following statements:

- [S1] $A^k = 0$ for some $k \geq 1$.
- [S2] A has two distinct real eigenvalues.
- [S3] A has rank 1.

Which one of the following is necessarily true?

- (a) Only [S1]
- (b) Only [S2]
- (c) Only [S2] and [S3]
- (d) Only [S3]

Options :

- 12820644875. A
- 12820644876. B
- 12820644877. C
- 12820644878. D

Question Number : 9 Question Id : 12820611358 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous map and $S = \left\{ \pi + \frac{(-1)^n}{2n+1} \mid n \in \mathbb{N} \right\} \cup (0, 1)$. Which one of the following is necessarily true?

- (a) $f((0, 1))$ is an open set
- (b) $f(\overline{S})$ is compact
- (c) $f(S)$ is compact
- (d) $f(S)$ is closed but not compact

Options :

12820644879. A

12820644880. B

12820644881. C

12820644882. D

Question Number : 10 Question Id : 12820611359 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a map that sends $(x, y) \in \mathbb{R}^2$ to $f(x, y)$. Which one of the following is necessarily true?

- (a) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0, 0)$, then f is continuous at $(0, 0)$
- (b) If $\frac{\partial f}{\partial x}$ exists at $(0, 0)$, then f is continuous separately in both variables x and y at $(0, 0)$
- (c) If f is continuous at $(0, 0)$, then $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0, 0)$
- (d) None the above

Options :

12820644883. A

12820644884. B

12820644885. C

12820644886. D

Question Number : 11 Question Id : 12820611360 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 5 Wrong Marks : 0

Let p be a prime number. Let $\sum_{n=1}^{p-1} \frac{1}{n} = \frac{\alpha}{\beta}$, where α, β are positive integers and are relatively prime. Which one of the following is necessarily true?

- (a) $p \mid \alpha$, if $p = 3, 5, 7$ and $p \nmid \alpha$, if $p \geq 11$
- (b) $p \mid \alpha$ for all odd prime numbers p
- (c) $\beta \equiv -1 \pmod{p}$, for all p
- (d) None of the above

Options :

12820644887. A

12820644888. B

12820644889. C

12820644890. D

PART 2

Section Id :	128206326
Section Number :	2
Section type :	Online
Mandatory or Optional:	Mandatory
Number of Questions:	15
Number of Questions to be attempted:	15
Section Marks:	45
Display Number Panel:	Yes
Group All Questions:	No

Sub-Section Number:	1
Sub-Section Id:	128206525
Question Shuffling Allowed :	Yes

Question Number : 12 Question Id : 12820611361 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Consider the permutation $\sigma = (1\ 2)(3\ 4)(4\ 5\ 6)(1\ 2\ 3)$ in S_7 . Then, the order of σ equals

- (a) 2
- (b) 3
- (c) 5
- (d) 6

Options :

- 12820644891. A
- 12820644892. B
- 12820644893. C
- 12820644894. D

Question Number : 13 Question Id : 12820611362 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

What is the volume of the region in the positive octant in \mathbb{R}^3 bounded by the planes $x = 0, y = 0, z = 0, x = 3$ and the parabolic cylinder $z = 4 - y^2$?

- (a) 4
- (b) 8
- (c) 16
- (d) ∞

Options :

- 12820644895. A
- 12820644896. B
- 12820644897. C
- 12820644898. D

Question Number : 14 Question Id : 12820611363 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

How many real roots does the polynomial $3X^5 + 9X + 7$ have?

(a) 1

(b) 3

(c) 5

(d) 0

Options :

12820644899. A

12820644900. B

12820644901. C

12820644902. D

Question Number : 15 Question Id : 12820611364 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

The value of $\lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^{2x}$ is

(a) 0

(b) 1

(c) e^2

(d) $+\infty$

Options :

12820644903. A

12820644904. B

12820644905. C

12820644906. D

Question Number : 16 Question Id : 12820611365 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let $m \equiv (20)^{18} \pmod{7}$. Then, $m \pmod{7}$ is equal to

(a) 6

(b) 2

(c) 1

(d) None of these

Options :

12820644907. A

12820644908. B

12820644909. C

12820644910. D

Question Number : 17 Question Id : 12820611366 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Consider

$$S = \left\{ 2\pi + (-1)^n \tan\left(\frac{(-1)^n}{2n+1}\right) \mid n \in \mathbb{N} \right\} \cup \{3 \sin x \mid x \in (0, \pi/2)\} \\ \cup \{4e^{5x} \mid x \in (-\infty, 0)\}.$$

Then, the set of limit points of S is

- (a) $(0, 4]$
- (b) $(0, 4] \cup \{2\pi\}$
- (c) $[0, 4] \cup \{2\pi\}$
- (d) $(-\infty, 4] \cup \{2\pi\}$

Options :

12820644911. A

12820644912. B

12820644913. C

12820644914. D

Question Number : 18 Question Id : 12820611367 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let a map $f : (-1, 1) \rightarrow \mathbb{R}$ be continuous at 0. Consider the following statements:

- [S1] $\lim_{x \rightarrow 0^+} f(x) = f(0) = \lim_{x \rightarrow 0^-} f(x)$
- [S2] $\sup\{f(x) \mid x \in (-1, 0)\} = f(0) = \inf\{f(x) \mid x \in (0, 1)\}$
- [S3] $\lim_{n \rightarrow \infty} f\left(\frac{(-1)^n}{2n+1}\right)$ exists and equals $f(0)$.
- [S4] $\sup\{f\left(\frac{1}{n}\right) \mid n \in \mathbb{N}\} = f(0) = \inf\{f\left(\frac{1}{n}\right) \mid n \in \mathbb{N}\}$

Which of the above statements are necessarily true?

- (a) Only [S1]
- (b) Only [S1] and [S3]
- (c) Only [S1], [S2] and [S3]
- (d) Only [S1], [S3] and [S4]

Options :

12820644915. A

12820644916. B

12820644917. C

12820644918. D

Question Number : 19 Question Id : 12820611368 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function. Which one of the following is necessarily true?

- (a) If $f'(0) > 0$, then $f(x) > f(0)$ for all $x \in (0, 1)$.
- (b) If $f'(0) < 0$, then $f(x) < f(0)$ for all $x \in (0, \delta)$ for some $\delta > 0$.
- (c) If $f'(0) = 0$, then $f(x) = f(0)$ for all $x \in (-\delta, \delta)$ for some $\delta > 0$.
- (d) If $f'(0) > 0$, then $f(0) > 0$.

Options :

12820644919. A

12820644920. B
12820644921. C
12820644922. D

Question Number : 20 Question Id : 12820611369 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let $S = \mathbb{Q} \cap [0,1]$. Let f be a bijection from \mathbb{N} to S . Which one of the following is necessarily true?

- (a) $\frac{1}{2}$ is a limit point but $\frac{1}{\sqrt{2}}$ is not a limit point of the sequence $\{f(n)\}_{n \geq 1}$.
(b) $\frac{1}{\sqrt{2}}$ is a limit point but $\frac{1}{2}$ is not a limit point of the sequence $\{f(n)\}_{n \geq 1}$.
(c) We can choose a bijection f from \mathbb{N} to S such that $\frac{1}{\sqrt{2}}$ is not a limit point of the sequence $\{f(n)\}_{n \geq 1}$.
(d) Both $\frac{1}{2}$ and $\frac{1}{\sqrt{2}}$ are limit points of the sequence $\{f(n)\}_{n \geq 1}$.

Options :

12820644923. A
12820644924. B
12820644925. C
12820644926. D

Question Number : 21 Question Id : 12820611370 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Which one of the following is true?

- (a) The number of groups of order 14 up to isomorphism is 2
(b) The number of groups of order 14 up to isomorphism is 3
(c) The number of groups of order 26 up to isomorphism is 1
(d) The number of groups of order 14 up to isomorphism is 1

Options :

12820644927. A
12820644928. B
12820644929. C
12820644930. D

Question Number : 22 Question Id : 12820611371 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Which one of the following is necessarily true?

- (a) If $f : \mathbb{Q} \rightarrow \mathbb{R}$ is an injective map, then $\overline{f(\mathbb{Q})}$ is uncountable
(b) There exists no surjective map from \mathbb{N} onto \mathbb{Q}
(c) There exists a surjective map from $\mathbb{Q} \times \mathbb{R}$ onto \mathbb{C}
(d) None of the above

Options :

12820644931. A
12820644932. B
12820644933. C
12820644934. D

Question Number : 23 Question Id : 12820611372 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Consider the statements:

[S1] $\{\sin(\frac{1}{n}) \mid n \geq 1\}$ is uncountable.

[S2] $\{e^{\frac{1}{n}} \mid n \geq 1\}$ is uncountable.

[S3] $\{\pi^n \mid n \geq 1\}$ is uncountable.

Which one of the following is the correct option?

- (a) None of [S1], [S2] and [S3] is true
- (b) All of [S1], [S2] and [S3] are true
- (c) Only [S1] is true
- (d) Only [S1] and [S2] are true

Options :

12820644935. A

12820644936. B

12820644937. C

12820644938. D

Question Number : 24 Question Id : 12820611373 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let $A \in M_2(\mathbb{R})$ such that $AA^T = A^T A = I$ and $\det(A) = 1$. Which one of the following is necessarily true?

- (a) $Av = v$ for some unit vector $v \in \mathbb{R}^2$
- (b) $Av \neq v$ for some unit vector $v \in \mathbb{R}^2$
- (c) $\|Av\| \neq 1$ for some unit vector $v \in \mathbb{R}^2$
- (d) None of the above

Options :

12820644939. A

12820644940. B

12820644941. C

12820644942. D

Question Number : 25 Question Id : 12820611374 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

Let r be a positive real number. Then, $\int_0^\infty \frac{dx}{1+e^{rx}}$ is equal to

- (a) $\frac{\log 2}{r}$
- (b) $\frac{1}{r}$
- (c) $\frac{1}{er}$
- (d) $\frac{1}{e^r}$

Options :

12820644943. A

12820644944. B

12820644945. C

12820644946. D

Question Number : 26 Question Id : 12820611375 Question Type : MCQ Option Shuffling : No Display Question Number : Yes
Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 3 Wrong Marks : 0

What is the value of $\int_0^1 \int_y^1 e^{-x^2} dx dy$?

- (a) Undefined
- (b) $\sqrt{\pi}$
- (c) $\frac{e+1}{2e}$
- (d) $\frac{e-1}{2e}$

Options :

12820644947. A

12820644948. B

12820644949. C

12820644950. D