National Testing Agency

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Topology

Group Number :	1
Group Id :	709597333
Group Maximum Duration :	0
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Revisit allowed for view? :	No
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Break time:	0
Group Marks:	100

Topology -1

Section Id :	709597432
Section Number :	1
Section type :	Online
Mandatory or Optional:	Mandatory
Number of Questions:	20
Number of Questions to be attempted:	20
Section Marks:	20
Display Number Panel:	Yes
Group All Questions:	No

Sub-Section Number:	1
Sub-Section Id:	709597536
Question Shuffling Allowed :	No

Question Number : 1 Question Id : 70959730202 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

If τ_1 and τ_2 are two topologies on a non-empty set then
(A) $\tau_1 \cap \tau_2$ is a topology
(B) $\tau_1 \cup \tau_2$ is a topology
(C) $\tau_1 \setminus \tau_2$ is a topology
(D) $\tau_2 \setminus \tau_1$ is a topology.

Question Number : 2 Question Id : 70959730203 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

A uniform continuous function
(A) maps Cauchy sequences to Cauchy sequences
(B) is bijective
(C) may not map convergent sequences to convergent sequences
(D) need not be continuous.

Question Number : 3 Question Id : 70959730204 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let A, B be connected subsets of $(\mathbb{R}, d)$ where d is the usual distance in $\mathbb{R}$. If $A \cap B \neq \emptyset$, then which the following set may not be connected.
(A) $A \cup B$
(B) $A \cap B$
(C) $A \setminus B$
(D) $A \times B$ in $\mathbb{R}^2$ (Euclidean distance)

Question Number : 4 Question Id : 70959730205 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let (X, d) be a metric space. Then
(A) Intersection of arbitrary collection of open sets in X is open in X
(B) Union of arbitrary collection of closed sets in X is closed in X
(C) At least one of the above is true.
(D) Each of the above is false

Question Number : 5 Question Id : 70959730206 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let $A = (0, 1) \subseteq \mathbb{R}$. Then
(A) Each point of A is a limit point of A
(B) 0 and 1 are limit points of A
(C) The set of limit points of A is $[0, 1]$.
(D) Nothing can be said about the limit points of A, because the underlying metric is not specified.

Question Number : 6 Question Id : 70959730207 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Which of the following metric space is complete?
(A) $(0, 1)$ with usual metric
(B) $\mathbb{Q}$ with usual metric
(C) $[0, 1]$ with usual metric
(D) $[0, 1] \setminus 0$ with usual metric

Question Number : 7 Question Id : 70959730208 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let A and B be any two subsets of a metric space (X, d). Then
(A) $A^o \cup B^o = (A \cup B)^o$
(B) $A^o \cup B^o \supseteq (A \cup B)^o$
(C) $A^o \cup B^o \subseteq (A \cup B)^o$
(D) None of these

Question Number : 8 Question Id : 70959730209 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let (X, d) be a connected metric space. If $f : X \longrightarrow \mathbb{R}$ (d usual) is a non-constant continuous function .Then, $f(X)$ is
(A) finite set
(B) countable set
(C) singleton set
(D) singleton set

Question Number : 9 Question Id : 70959730210 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let X be the set of all real sequences $x = (x_n)$. Consider the metric d defined by

$$\begin{aligned} d(x, y) &= \quad 0 \qquad \text{if } x = y \\ &= \frac{1}{\min\{i : x_i \neq y_i\}} \quad \text{if } x \neq y \end{aligned}$$

where $x = (x_n), y = (y_n) \in X$. Then for distinct sequences $x, y, z \in X$
(A) $d(x, z) \leq d(x, y) + d(y, z)$ and the equality may hold.
(B) $d(x, z) \leq \max\{d(x, y), d(y, z)\}$
(C) $d(x, z) \geq \max\{d(x, y), d(y, z)\}$
(D) None of the above.

Question Number : 10 Question Id : 70959730211 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Which of the spaces X and Y are homeomorphic?
(A) $X = \mathbb{R}$, $Y = [0, 1)$.
(B) $X = \mathbb{R}$, $Y = [0, 1]$.
(C) $X = \mathbb{R}$, $Y = (0, 1)$.
(D) None of the above.

Question Number : 11 Question Id : 70959730212 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Which of the following metric spaces are not complete?
(A) $\mathbb{R}$ with usual metric
(B) $\mathbb{R}^n$ with euclidean metric
(C) Q with usual metric.
(D) $[0, 1]$ with usual metric

Question Number : 12 Question Id : 70959730213 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

The Cantor set is
(A) compact with usual topology induced by $\mathbb{R}$.
(B) compact with discrete topology.
(C) not compact with respect to any topology.
(D) None of these.

Question Number : 13 Question Id : 70959730214 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 1 Wrong Marks : 0

Which of the following maps is an isometry?
(A) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + 1$ ($\mathbb{R}$ equipped with usual metric)
(B) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x$ ($\mathbb{R}$ equipped with usual metric)
(C) Any Lipschitz map
(D) The map $f : (X, d) \to (X, d')$ defined by $f(x) = x$ where d is a discrete metric.

Question Number : 14 Question Id : 70959730215 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 1 Wrong Marks : 0

Let A, B be compact subsets of $(\mathbb{R}, d)$, (d being usual). Then which of the following set is not compact:
(A) $A \times B$ in $(\mathbb{R}^2, d)$, d being Euclidean
(B) $A \cup B$ in $\mathbb{R}$
(C) $A \cap B$ in $\mathbb{R}$ (provided $A \cap B \neq \emptyset$).
(D) $A \setminus B$ in $\mathbb{R}$ (provided $A \setminus B \neq \emptyset$).

Question Number : 15 Question Id : 70959730216 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 1 Wrong Marks : 0

Which of the following is not a Baire space?
(A) $\mathbb{R}$.
(B) $\mathbb{R}^2$.
(C) Cantor set.
(D) $\mathbb{Q}$.

Question Number : 16 Question Id : 70959730217 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 1 Wrong Marks : 0

The *Bolzanno-Weierstrass' Theorem* states that:
(A) every bounded sequence in $\mathbb{R}^n$ with usual metric has a convergent subsequence.
(B) every Cauchy sequence in a metric space (X, d) is convergent
(C) every bounded sequence in $\mathbb{R}^n$ with usual metric is convergent
(D) every convergent sequence in a metric space (X, d) is vague statement Cauchy

Question Number : 17 Question Id : 70959730218 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 1 Wrong Marks : 0

The subset $\{0\} \times (a, b)$ of $\mathbb{R}^2$ is:
(A) Neither open nor closed
(B) Open
(C) Closed
(D) Both open and closed.

Question Number : 18 Question Id : 70959730219 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

Let X, Y be topological spaces and $p : X \longrightarrow Y$ be a quotient map. Let A be subspace of X; let $q : A \longrightarrow p(A)$ be a map obtained by restricting p . Then
(A) q is quotient if A is open
(B) q is quotient if A is closed
(C) q is quotient if p is open
(D) none of the above

Question Number : 19 Question Id : 70959730220 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

The unit circle S^1 with finite complement topology is
(A) Hausdorff
(B) Not Hausdorff
(C) Normal
(D) None of the above.

Question Number : 20 Question Id : 70959730221 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 1 Wrong Marks : 0

If X is an ordered set in the order topology and if Y is an interval or a ray in X
(A) Then the subspace topology and order topology on Y are same.
(B) Then the subspace topology and order topology on Y are different.
(C) only if Y is a singleton set.
(D) None of these.

Topology -2

Section Id :	709597433
Section Number :	2
Section type :	Offline
Mandatory or Optional:	Mandatory
Number of Questions:	10
Number of Questions to be attempted:	10
Section Marks:	30
Display Number Panel:	Yes
Group All Questions:	No

Sub-Section Number:	1
Sub-Section Id:	709597537
Question Shuffling Allowed :	No

Question Number : 21 Question Id : 70959730222 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

Prove that a subset P of a space X is dense in X iff for every non-empty open subset Q of X, $P \cap Q \neq \phi$.

Question Number : 22 Question Id : 70959730223 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

Let (X, d), and (Y, d) be metric spaces. Show that a function $f : X \to Y$ is continuous iff for any open subset V of Y, the subset $f^{-1}(V)$ is open in X.

Question Number : 23 Question Id : 70959730224 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

The closure of any set is the union of the set and the set of its accumulation points. Prove or disprove.

Question Number : 24 Question Id : 70959730225 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

Let (X, d) be a metric space. For any two distinct elements $p, q \in X$ show that there exists $r > 0$ such that $B(p, r) \cap B(q, r) = \emptyset$

Question Number : 25 Question Id : 70959730226 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

Let (X, d) be a metric space and let (x_n) be a sequence in X. Then show that $(x_n) \to x$ in X if and only if every subsequence $(x_{n_k}) \to x$ in X.

Question Number : 26 Question Id : 70959730227 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

If X has the discrete topology and Y is any topological space, then show that any function $f : X \to Y$ is continuous.

Question Number : 27 Question Id : 70959730228 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

If X and Y are separable then show that $X \times Y$ is also separable.

Question Number : 28 Question Id : 70959730229 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

Show that Hausdroff condition is a hereditary property.

Question Number : 29 Question Id : 70959730230 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

If X is a compact space and $A \subset X$ is a closed subset, the show that A is compact.

Question Number : 30 Question Id : 70959730231 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 3

Let X be sequentially compact. Show that for every $\epsilon > 0$ there exists a finite covering of X by ϵ-balls.

Topology -3

Section Id :	709597434
Section Number :	3
Section type :	Offline
Mandatory or Optional:	Mandatory
Number of Questions:	7
Number of Questions to be attempted:	5
Section Marks:	50
Display Number Panel:	Yes
Group All Questions:	No

Sub-Section Number:	1
Sub-Section Id:	709597538
Question Shuffling Allowed :	No

Question Number : 31 Question Id : 70959730232 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Let $(X, d_1), (Y, d_2), (Z, d_3)$ be metric spaces. Show that
(i) If $f : X \to Y$ is continuous and $A \subset X$, then $f|_A : A \to Y$ is also continuous.
(ii) If $f : X \to Y, g : Y \to Z$ are continuous, then $g \circ f$ is also continuous.

Question Number : 32 Question Id : 70959730233 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Prove that $X \times Y$ is compact iff both X and Y are compact.

Question Number : 33 Question Id : 70959730234 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Show that the projection of a product space into each of its coordinate spaces is open.

Question Number : 34 Question Id : 70959730235 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Show that the following are equivalent on a space X:
(i) X is connected
(ii) The only subsets of X which are open and closed are X and ϕ
(iii) X cannot be expressed as union of two disjoint non-empty open sets
(iv) There is no onto continuous function from X to a discrete space which contains more than one point.

Question Number : 35 Question Id : 70959730236 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Suppose (X, d) is a metric space and suppose $A \subseteq X$. Then prove the following:
(i) $A^o \subseteq A$
(ii) A^o is open
(iii) A is open if and only if $A = A^o$

Question Number : 36 Question Id : 70959730237 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Let X be a normal space. Let A be a closed subspace of X. Prove that
(i) Any continuous map of A into the closed interval $[a, b]$ of $\mathbb{R}$ may be extended to a continuous map of all X into $[a, b]$.

(ii) Any continuous map of A into $\mathbb{R}$ may be extended to a continuous map of all of X into $\mathbb{R}$.

Question Number : 37 Question Id : 70959730238 Question Type : SUBJECTIVE Display Question Number : Yes
Correct Marks : 10

Let $p : X \longrightarrow Y$ be a quotient map. Let Z be a space and $g : X \longrightarrow Z$
be a map that is constant on each set $p^{-1}(\{y\})$, for $y \in Y$

Then g induces a map $f : Y \longrightarrow Z$ such that $f \circ p = g$. The induced map f is continuous if and only if g is continuous; f is quotient if and only if g is quotient.