National Testing Agency

Question Paper Name:	Fourier Analysis and its Applications 16th February 2020 Shift 1
Subject Name:	Fourier Analysis and its Applications
Creation Date:	2020-02-16 12:58:20
Duration:	180
Total Marks:	100
Display Marks:	Yes

Fourier Analysis and its Applications

Group Number :	1
Group Id :	28860735
Group Maximum Duration :	0
Group Minimum Duration :	120
Show Attended Group? :	No
Edit Attended Group? :	No
Break time:	0
Group Marks:	100
Is this Group for Examiner?:	No

Fourier Analysis and its Applications

Section Id :	28860738
Section Number :	1
Section type :	Online
Mandatory or Optional:	Mandatory
Number of Questions:	70
Number of Questions to be attempted:	50
Section Marks:	100

Sub-Section Number:	1
Sub-Section Id:	28860741
Question Shuffling Allowed :	Yes

Question Number : 1 Question Id : 2886073175 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Consider the 2π periodic function

$$f(x) = \sum_{-\infty}^{\infty} \exp(-(x + 2\pi n)^2)$$

The constant term in the Fourier series expansion of $f(x)$

equals:

(A) 1
(B) $\frac{1}{2\sqrt{\pi}}$
(C) $\sqrt{2\pi}$
(D) 2π

Options :

28860712673. 1

28860712674. 2

28860712675. 3

28860712676. 4

Question Number : 2 Question Id : 2886073176 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The value of $1 + \frac{1}{3^2} + \ + \frac{1}{5^2} + \cdots$ equals:

(A) $\pi^2/6$
(B) $\pi^2/8$
(C) $\pi^2/16$
(D) $\pi^2/18$

Options :

28860712677. 1

28860712678. 2

28860712679. 3

28860712680. 4

Question Number : 3 Question Id : 2886073177 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Coefficients a_n, b_n in the Fourier series expansion of a 2π periodic function behave like n^{-1-c} where c is a positive number. Then which of the following is true?

(A) The series converges uniformly and absolutely
(B) The series converges uniformly and but not absolutely
(C) The series converges absolutely but not uniformly
(D) The series converges neither uniformly nor absolutely

Options :

28860712681. 1

28860712682. 2

28860712683. 3

28860712684. 4

Question Number : 4 Question Id : 2886073178 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Supply the value of c in Rodrigues formula for the Legendre polynomial

$P_n(x) = c\frac{d^n}{dx^n}(x^2-1)^n$ equals:

(A) 1
(B) 2^n
(C) $\frac{1}{2^n n!}$
(D) $2^n n!$

Options :

28860712685. 1

28860712686. 2

28860712687. 3

28860712688. 4

Question Number : 5 Question Id : 2886073179 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

If $f(x)$ is in the Schwartz class and is an even function then the Fourier transform of $\hat{f}(\xi)$ equals:

(A) $f(x)$
(B) $-f(x)$
(C) $\pi f(x)$
(D) $2\pi f(x)$

Options :

28860712689. 1

28860712690. 2

28860712691. 3

28860712692. 4

Question Number : 6 Question Id : 2886073180 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Given that $f(x) = \frac{1}{1+x^2}$ what is the value of $\hat{f}(0)$?

A $\pi/2$

B π

C 2π

D 1

Options :

28860712693. 1

28860712694. 2

28860712695. 3

28860712696. 4

Question Number : 7 Question Id : 2886073181 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is the expression for the Dirichlet kernel $D_n(x)$

A $\frac{1}{2\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$

B $\frac{1}{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$

C $\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$

D $\frac{1}{2}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$

Options :

28860712697. 1

28860712698. 2

28860712699. 3

28860712700. 4

Question Number : 8 Question Id : 2886073182 Question Type : MCQ Option Shuffling : No

Correct Marks : 2 Wrong Marks : 0

Which of the following statement about the Volterra operator $T : L^2[0,1] \to L^2[0,1]$ is false?

A It is a compact operator

B It is not a self-adjoint operator

C It has no eigen-values

D 0 is in the spectrum of this operator.

Options :

28860712701. 1

28860712702. 2

28860712703. 3

28860712704. 4

Question Number : 9 Question Id : 2886073183 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Fejer's theorem asserts that for a continuous 2π periodic function $f(x)$, the nth partial sum $S_n(f,x)$

A Converges uniformly to $f(x)$

B Converges pointwise but not uniformly to $f(x)$

C The Cesaro means of $S_n(f,x)$ converges uniformly to $f(x)$

D The Cesaro means of $S_n(f,x)$ converges pointwise but not uniformly to $f(x)$

Options :

28860712705. 1

28860712706. 2

28860712707. 3

28860712708. 4

Question Number : 10 Question Id : 2886073184 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is true?

A The Fejer and Poisson kernels are positive kernels

B The Dirichlet and Poisson kernels are positive kernels

C The Dirichlet and Fejer kernels are positive kernels

D The Dirichlet, Poisson and Fejer kernels are positive kernels

Options :

28860712709. 1

28860712710. 2

28860712711. 3

28860712712. 4

Question Number : 11 Question Id : 2886073185 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Use the function $f(x)$ which equals 1 on the interval $[-1, 1]$ and zero outside this interval to compute the integral $\int_{-\infty}^{\infty} \left(\frac{\sin\xi}{\xi}\right)^2 \, d\xi$

A $\frac{1}{4\pi}$

B $\frac{1}{2\pi}$

C 2π

D 4π

Options :

28860712713. 1

28860712714. 2

28860712715. 3

28860712716. 4

Question Number : 12 Question Id : 2886073186 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Parseval formula applied to the function $f(x) = |x|^2$ on $[-\pi, \pi]$ gives the value of $1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$ as being equal to

A $\frac{\pi^4}{64}$

B $\frac{\pi^4}{80}$

C $\frac{\pi^4}{90}$

D $\frac{\pi^4}{96}$

Options :

28860712717. 1

28860712718. 2

28860712719. 3

28860712720. 4

Question Number : 13 Question Id : 2886073187 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

The polynomials $e^{\frac{x^2}{2}}\left(\frac{d}{dx}\right)^n e^{\frac{-x^2}{2}}$ satisfies which of the following equations?

A $(1-x^2)y''-2xy'+n(n+1)y=0$

B $(1-x^2)y''-xy'+n^2y=0$

C $y''-2xy'+2ny=0$

D $(1-x^2)y''+2xy'+n(n+1)y=0$

Options :

28860712721. 1

28860712722. 2

28860712723. 3

28860712724. 4

Question Number : 14 Question Id : 2886073188 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Suppose $P_n(x)$ is the $n-th$ Legendre polynomial then the value of $P_n'(1)$ equals

A $n(n+1)/2$

B $n(n-1)/2$

C $n(n+1)$

D $n(n-1)$

Options :

28860712725. 1

28860712726. 2

28860712727. 3

28860712728. 4

Question Number : 15 Question Id : 2886073189 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

The Fourier transform of the function $f(x) = e^{-|x|}$ equals:

A $2/(1+\xi^2)$

B $1/(1+\xi^2)$

C $1/2(1+\xi^2)$

D $1/4(1+\xi^2)$

Options :

28860712729. 1

28860712730. 2

28860712731. 3

28860712732. 4

Question Number : 16 Question Id : 2886073190 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Which of the following is not an eigen-value of the Fourier transform as an operator on $L^2(\mathbb{R})$

A $\sqrt{\pi}$

B $\sqrt{2\pi}$

C $i\sqrt{2\pi}$

D $-i\sqrt{2\pi}$

Options :

28860712733. 1

28860712734. 2

28860712735. 3

28860712736. 4

Question Number : 17 Question Id : 2886073191 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Which of the following is the solution of the heat equation $u_t = u_{xx}$ with $u(x,0) = f(x)$? Here $f(x)$ is in the Schwartz class.

A $(4\pi t)^{-1} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t)\, dy$

B $(4\pi t)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t)\, dy$

C $(\pi t)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t)\, dy$

D $(4t)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t)\, dy$

Options :

28860712737. 1

28860712738. 2

28860712739. 3

28860712740. 4

Question Number : 18 Question Id : 2886073192 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

What is the value of $\int_{-\infty}^{\infty} \frac{\sin x}{x}\, dx$? Hint: Think of using the Laplace transform of $\int_{-\infty}^{\infty} \frac{\sin tx}{x}\, dx$ with respect to t.

A $\frac{\pi}{2}$

B 2π

C $\sqrt{\pi}$

D π

Options :

28860712741. 1

28860712742. 2

28860712743. 3

28860712744. 4

Question Number : 19 Question Id : 2886073193 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Consider the function $f(x) = 1 + \cos 3x + \sin^5 x$. The coefficients a, b of $\cos 6x$ and $\sin 5x$ in the Fourier series for $f(x)$ are respectively

A $a = 1, b = 1/16$

B $a = 1, b = 1/8$

C $a = 0, b = \frac{1}{16}$

D $a = 0, b = \frac{1}{8}$

Options :

28860712745. 1

28860712746. 2

28860712747. 3

28860712748. 4

Question Number : 20 Question Id : 2886073194 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Consider the function function $f(x) = x/|x|$, $f(0) = 0$ on $[-\pi, \pi]$. Supply the value of the constant c in the following Fourier expansion

$$f(x) = \frac{c}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$$

A 2.

B 4

C 1/2

D 1/4

Options :

28860712749. 1

28860712750. 2

28860712751. 3

28860712752. 4

Question Number : 21 Question Id : 2886073195 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

For which of the following kernels $k(x, y)$ is the associated integral operator on $L^2[0, 1]$ compact and self adjoint?

A $k(x, y) = 1$, if $x < y$ and $k(x, y) = 0$, if $x > y$

B $k(x, y) = 0$, if $x < y$ and $k(x, y) = 1$, if $x > y$

C $k(x, y) = \frac{\sin(2x-y)}{2x-y}$

D $k(x, y) = \frac{\sin(x-y)}{x-y}$

Options :

28860712753. 1

28860712754. 2

28860712755. 3

28860712756. 4

Question Number : 22 Question Id : 2886073196 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The Lebesgue constants are the numbers $L_n = \int_0^{\pi} |D_n(t)| dt$ where $D_n(t)$ is the Dirichlet kernel. Which of the following is TRUE ?

A $\sum_0^{\infty} L_n$ converges.

B $\sum_0^{\infty} L_n^{-1}$ converges

C $\sum_0^{\infty} (nL_n)^{-2}$ converges

D $\sum_0^{\infty} (nL_n)^{-1}$ converges

Options :

28860712757. 1

28860712758. 2

28860712759. 3

28860712760. 4

Question Number : 23 Question Id : 2886073197 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Consider the solution $u(x, y)$ of $\Delta u = 0$ on $\{(x, y): x^2 + y^2 < 1\}$ and $u(\cos\theta, \sin\theta) = |\cos\theta|$. Then the value of $u(0, 0)$ equals:

A $\frac{\pi}{2}$

B $1/\pi$

C $2/\pi$

D π

Options :

28860712761. 1

28860712762. 2

28860712763. 3

28860712764. 4

Question Number : 24 Question Id : 2886073198 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The value of $\int_0^{2\pi} \frac{\left(1-\frac{1}{9}\right)d\theta}{\left(1+\frac{1}{9}-\frac{2}{3}\cos\theta\right)}$ equals:

A $\frac{\pi}{4}$

B $\frac{\pi}{2}$

C π

D 2π

Options :

28860712765. 1

28860712766. 2

28860712767. 3

28860712768. 4

Question Number : 25 Question Id : 2886073199 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

S is a closed bounded subset of $C[0,1]$ endowed with the supremum norm. To apply the Ascoli-Arzela theorem which additional conditions are needed on S

A S consists of non-negative functions

B S is equi-continuous

C Every member of S is Lipschitz continuous

D No additional hypothesis is needed.

Options :

28860712769. 1

28860712770. 2

28860712771. 3

28860712772. 4

Question Number : 26 Question Id : 2886073200 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Given that $x^4 = c_4P_4(x) + c_3 3(x) + \cdots + c_0P_0(x)$ where $P_n(x)$ is the $n-th$ Legendre polynomial, the value of c_4 equals:

A 8/15

B 8/25

C 8/35

D 8/45

Options :

28860712773. 1

28860712774. 2

28860712775. 3

28860712776. 4

Question Number : 27 Question Id : 2886073201 Question Type : MCQ Option Shuffling : No

Correct Marks : 2 Wrong Marks : 0

What is the value of $2\int_0^1 P_4(x)dx$? Here $P_n(x)$ is the n-th Legendre polynomial.

A 0

B 3/16

C $-3/16$

D 3/8

Options :

28860712777. 1

28860712778. 2

28860712779. 3

28860712780. 4

Question Number : 28 Question Id : 2886073202 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Supply the constant a in the following formula:

$$a\int_{-\pi}^{\pi}|f(x)|^2\,dx = |a_0|^2 + \frac{1}{2}\sum_1^{\infty}(|a_n|^2 + |b_n|^2)$$

A $1/\pi$

B $2/\pi$

C 2

D $1/(2\pi)$

Options :

28860712781. 1

28860712782. 2

28860712783. 3

28860712784. 4

Question Number : 29 Question Id : 2886073203 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The constant term in the Fourier series expansion of $f(x) = x^4$ on the interval $[-\pi, \pi]$ equals:

A $\pi^3/5$

B $\pi^3/10$

C $\pi^4/5$

D $\pi^4/10$

Options :

28860712785. 1

28860712786. 2

28860712787. 3

28860712788. 4

Question Number : 30 Question Id : 2886073204 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

What is the value of $\lim_{n\to\infty} \frac{\sqrt[n]{(2n)!}}{n^2}$

A $1/e^2$

B $4/e^2$

C $\sqrt{2\pi}/e^2$

D $\sqrt{32\pi}/e^2$

Options :

28860712789. 1

28860712790. 2

28860712791. 3

28860712792. 4

Question Number : 31 Question Id : 2886073205 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

The $n-th$ Legendre polynomial is $P_n(x)$. Then the value of $P_5'(0)$ equals

A $\frac{15}{4}$

B $\frac{15}{8}$

C $\frac{15}{16}$

D $\frac{15}{36}$

Options :

28860712793. 1

28860712794. 2

28860712795. 3

28860712796. 4

Question Number : 32 Question Id : 2886073206 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Given that $f(x) = (1+x^2)^{-2}$ what is the value of $\hat{f}(0)$? Here $\hat{f}$ denotes the Fourier transform of f

A $\frac{\pi}{2}$

B π

C 2π

D 4π

Options :

28860712797. 1

28860712798. 2

28860712799. 3

28860712800. 4

Question Number : 33 Question Id : 2886073207 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is FALSE about the Fourier transform as an operator from $L^2(\mathbb{R})$ to itself

A It is invertible

B It is bounded

C It is unitary

D It is self-adjoint

Options :

28860712801. 1

28860712802. 2

28860712803. 3

28860712804. 4

Question Number : 34 Question Id : 2886073208 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The Lebesgue constants are the numbers $L_n = \int_0^{\pi} |D_n(t)|dt$ where $D_n(t)$ is the Dirichlet kernel. Which of the following is FALSE ?

A $\sum_2^{\infty} \frac{1}{n(\log L\_n)}$ diverges

B $\sum_2^{\infty} \frac{1}{n^2(\log L\_n)}$ converges

C $\sum_2^{\infty} \frac{1}{n^2(\log L\_n)}$ diverges

D $\sum_2^{\infty} \frac{1}{n(\log L\_n)^2}$ diverges

Options :

28860712805. 1

28860712806. 2

28860712807. 3

28860712808. 4

Question Number : 35 Question Id : 2886073209 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Fourier transform of e^{-x^2} is $ae^{-x^2/4}$. Then the value of a equals

A $\sqrt{\pi}$

B $\sqrt{2\pi}$

C $2\sqrt{\pi}$

D $4\sqrt{\pi}$

Options :

28860712809. 1

28860712810. 2

28860712811. 3

28860712812. 4

Question Number : 36 Question Id : 2886073210 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Which of the following is a trigonometric polynomial on $[-\pi, \pi]$

A $\sin(\frac{x}{5})$

B $\sin^5 x$

C $\sin^{\frac{1}{5}} x$

D $\sin\sqrt{5}x$

Options :

28860712813. 1

28860712814. 2

28860712815. 3

28860712816. 4

Question Number : 37 Question Id : 2886073211 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Which of the following is FALSE?

A Trigonometric polynomials are dense in $L^2[-\pi, \pi]$ with respect to L^2 norm

B Trigonometric polynomials are dense in $L^1[-\pi, \pi]$ with respect to L^1 norm

C Trigonometric polynomials are dense in $C[-\pi, \pi]$ with supremum

D Trigonometric polynomials are dense in the space of all continuous even functions with supremum norm

Options :

28860712817. 1

28860712818. 2

28860712819. 3

28860712820. 4

Question Number : 38 Question Id : 2886073212 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Let $f(x) = (1-x^2)^{-1/2}$ on $(-1,1)$ and zero outside. The Fourier transform of $f(x)$ Is

A $2\pi J_0(\xi)$

B $\pi J_0(\xi)$

C $\pi J_0(\xi)/2$

D $J_0(\xi)$

Options :

28860712821. 1

28860712822. 2

28860712823. 3

28860712824. 4

Question Number : 39 Question Id : 2886073213 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Given that $f(x) = \sin^2 x$ on $[-\pi, \pi]$ and zero outside $[-\pi, \pi]$ and $g(x) = \cos^2 x$ on $[-\pi, \pi]$ and zero outside $[-\pi, \pi]$. Then what is the value of $\int_{-\infty}^{\infty} f * g(x)dx$? Note $f * g$ is the convolution of f and g

A π^2

B $2\pi^2$

C 2π

D π

Options :

28860712825. 1

28860712826. 2

28860712827. 3

28860712828. 4

Question Number : 40 Question Id : 2886073214 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Given that the Fourier transform of a Schwartz function $f(x)$ is $F(\xi)$.

The Fourier transform of $F'(\xi)$ equals

A $2\pi i\, f(-x)$

B $2\pi i\, xf(x)$

C $2\pi i\, xf(x)$

D $2\pi i\, xf(-x)$

Options :

28860712829. 1

28860712830. 2

28860712831. 3

28860712832. 4

Question Number : 41 Question Id : 2886073215 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

A sequence of functions $f_n(x)$ in $L^2[0, 1]$ converges in norm then which of the following is false

A It converges weakly

B It is norm bounded

C The sequence of arithmetic means converges weakly

D Converges pointwise

Options :

28860712833. 1

28860712834. 2

28860712835. 3

28860712836. 4

Question Number : 42 Question Id : 2886073216 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is an odd function?

A $J_1'(x)$

B $P_5'(x)$

C Fourier transform of an odd function

D Fourier transform of an even function

Options :

28860712837. 1

28860712838. 2

28860712839. 3

28860712840. 4

Question Number : 43 Question Id : 2886073217 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Which of the following is an immediate consequence of Luzin's theorem?

A $C[0,1]$ is dense in $L^2[0,1]$ with respect to L^1 norm

B Polynomials are dense in $C[0,1]$

C Polynomials are dense in $L^1[0,1]$

D $C[0,1]$ is dense in $L^\infty[0,1]$ with respect to L^∞ norm

Options :

28860712841. 1

28860712842. 2

28860712843. 3

28860712844. 4

Question Number : 44 Question Id : 2886073218 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Fourier transform of a radial function is

A Always radial and given by a sine transform in two dimensions

B Always radial and given by a sine transform in three dimensions

C Always radial and given by a Bessel transform in three dimensions

D Always radial and given by a cosine transform in two dimensions

Options :

28860712845. 1

28860712846. 2

28860712847. 3

28860712848. 4

Question Number : 45 Question Id : 2886073219 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

For the heat equation $u_t - u_{xx} = 0$ with initial condition $u(x, 0) = f(x)$. Assume that $f(x)$ is positive on $(-1, 1)$ and zero outside. Then

A $u(x, t)$ is positive for all $x \in \mathbb{R}$ and for all positive t

B u(x, t) is positive only for all x∈(-1, 1) and for all positive t

C $u(x, t)$ is positive only for all $x \in (-2, 2)$ and for all positive t

D $u(x, t)$ is positive only for all $x \in (0, \infty)$ and for all positive t

Options :

28860712849. 1

28860712850. 2

28860712851. 3

28860712852. 4

Question Number : 46 Question Id : 2886073220 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

For the wave equation $u_{tt} - u_{xx} = 0$ with initial conditions in the Schwartz class, assume that $E(t)$ is the total energy and $\Pi(t)$ is the kinetic energy. Then, which of the following is true?

A $\Pi(t)$ is constant in time

B $\Pi(t)$ necessarily decays to zero as time approaches infinity

C $\Pi(t)$ approaches $E(0)/2$ as time goes to infinity

D $\Pi(t)$ approaches $E(0)$ as time goes to infinity

Options :

28860712853. 1

28860712854. 2

28860712855. 3

28860712856. 4

Question Number : 47 Question Id : 2886073221 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The sum $1 + 2\cos x + 2\cos 2x - \cdots + 2\cos nx$ equals:

A $\frac{\sin(nx+x)}{\sin\frac{x}{2}}$

B $\frac{\sin\left(nx+\frac{x}{2}\right)}{\sin\frac{x}{2}}$

C $\frac{\sin\left(nx+\frac{x}{2}\right)}{\sin x}$

D $\frac{\sin(nx)}{\sin x}$

Options :

28860712857. 1

28860712858. 2

28860712859. 3

28860712860. 4

Question Number : 48 Question Id : 2886073222 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is false about the $n-th$ Legendre polynomial?

A It has a double root at a point in $(-1,1)$

B It has n distinct roots in $(-1,1)$

C It does not vanish at 1

D For odd n it vanishes at the origin

Options :

28860712861. 1

28860712862. 2

28860712863. 3

28860712864. 4

Question Number : 49 Question Id : 2886073223 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Suppose T_n is a sequence of continuous linear maps from a Banach space X into $\mathbb{R}$ such that for each $x \in X$ the sequence $T_n(x)$ is bounded then which of the following is false?

A $||T_n||$ is bounded

B $\left|||T_n(x)|\right|$ has a bound independent of x for all $\left|||x|\right| \leq 1$

C $||T_n||$ converges

D $||T_n||$ has a convergent subsequence.

Options :

28860712865. 1

28860712866. 2

28860712867. 3

28860712868. 4

Question Number : 50 Question Id : 2886073224 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The Banach Steinhaus's theorem implies

A Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series converges uniformly

B Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series converges pointwise

C Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series diverges at a specific point.

D Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series diverges everywhere.

Options :

28860712869. 1

28860712870. 2

28860712871. 3

28860712872. 4

Question Number : 51 Question Id : 2886073225 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is not a trigonometric polynomial on $[-\pi, \pi]$

A $\cos 3x$

B $\cos^{1/3} x$

C $\cos^3 x$

D $3\cos x$

Options :

28860712873. 1

28860712874. 2

28860712875. 3

28860712876. 4

Question Number : 52 Question Id : 2886073226 Question Type : MCQ Option Shuffling : No

Correct Marks : 2 Wrong Marks : 0

Supply the value of the constant c in the following formula (where $f(x)$ is in the Schwartz's class) $\hat{\hat{f}}(x) = 2\pi f(cx)$

A -1

B 1

C -2

D 2

Options :

28860712877. 1

28860712878. 2

28860712879. 3

28860712880. 4

Question Number : 53 Question Id : 2886073227 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following about the Bessel's function $J_p(z)$ is true?

A It is an entire function of z for any value of p

B It is an entire function of z for no value of p

C It is an entire function of z when $p = k + \frac{1}{2}$ where k is a natural number

D It is an entire function of z for any integer value of p

Options :

28860712881. 1

28860712882. 2

28860712883. 3

28860712884. 4

Question Number : 54 Question Id : 2886073228 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The Weierstrass's approximation theorem implies

A The separability of $C[0, 1]$

B The separability of $L^{\infty}[0, 1]$

C That $C[0, 1]$ is not separable

D That $L^1[0, 1]$ is not separable.

Options :

28860712885. 1

28860712886. 2

28860712887. 3

28860712888. 4

Question Number : 55 Question Id : 2886073229 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is the Bessel's equation of order p?

A $x^2y'' - xy' + (x^2 + p^2) = 0$

B $x^2y'' - xy' + (x^2 - p^2) = 0$

C $x^2y'' + xy' + (x^2 + p^2) = 0$

D $x^2y'' + xy' + (x^2 - p^2) = 0$

Options :

28860712889. 1

28860712890. 2

28860712891. 3

28860712892. 4

Question Number : 56 Question Id : 2886073230 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

For vectors v and w in a Hilbert space, given that $||v|| = 4$, $||w|| = 5$ and $||w + v|| = 3$. Then what is the value of $||w - v||$?

A $\sqrt{63}$

B $\sqrt{69}$

C $\sqrt{71}$

D $\sqrt{73}$

Options :

28860712893. 1

28860712894. 2

28860712895. 3

28860712896. 4

Question Number : 57 Question Id : 2886073231 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

If $F(\xi)$ is the Fourier transform of $f(x)$ then the Fourier transform of $f'(x)$ equals:

A $\xi F(\xi)$

B $-\xi F(\xi)$

C $i\xi F(\xi)$

D $-i\xi F(\xi)$

Options :

28860712897. 1

28860712898. 2

28860712899. 3

28860712900. 4

Question Number : 58 Question Id : 2886073232 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

If $F(\xi)$ is the Fourier transform of $f(x)$ then the Fourier transform of $ixf(x)$ equals:

A $-iF(\xi)$

B $iF(\xi)$

C $-F(\xi)$

D $F(\xi)$

Options :

28860712901. 1

28860712902. 2

28860712903. 3

28860712904. 4

Question Number : 59 Question Id : 2886073233 Question Type : MCQ Option Shuffling : No Correct Marks : 2 Wrong Marks : 0

Which of the following is false?

A $L^1[-\pi,\pi]$ with respect to L^1 norm has uncountable Hamel basis

B $L^1[-\pi,\pi]$ with respect to L^1 norm has countable Hamel basis

C The set of all polynomials has countable Hamel basis

D $L^\infty[-\pi,\pi]$ with respect to L^∞ norm has uncountable Hamel basis

Options :

28860712905. 1

28860712906. 2

28860712907. 3

28860712908. 4

Question Number : 60 Question Id : 2886073234 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Let $y(x)$ be a solution of the differential equation $y'' + x^2 y = 0$. Then which of the following is true?

A The zeros of $y(x)$ form a finite set

B The zeros of $y(x)$ are equally spaced

C The zeros (z_n) of $y(x)$ are countable discrete increasing sequence and $|z_n - z_{n-1}|$ tends to zero as n tends to infinity.

D The zeros of $y(x)$ have a finite limit point

Options :

28860712909. 1

28860712910. 2

28860712911. 3

28860712912. 4

Question Number : 61 Question Id : 2886073235 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is an essential hypothesis on a metric space in order to apply the Baire Category theorem?

A X is complete

B X is locally connected

C X is connected

D X is separable

Options :

28860712913. 1

28860712914. 2

28860712915. 3

28860712916. 4

Question Number : 62 Question Id : 2886073236 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Given that F is the Fourier transform of a Schwartz function f

Supply the value of c in the following formula

$$c \int_{-\infty}^{\infty} |f(x)|^2 \, dx = \int_{-\infty}^{\infty} |F(\xi)|^2 \, d\xi$$

A π^2

B $\sqrt{2\pi}$

C 2π

D $1/\sqrt{2\pi}$

Options :

28860712917. 1

28860712918. 2

28860712919. 3

28860712920. 4

Question Number : 63 Question Id : 2886073237 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Given that $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ which of the following does not always hold? Here $F(\xi)$ is the Fourier transform of f

A $F(\xi)$ is uniformly continuous

B $F(\xi) \in L^1(\mathbb{R})$

C $F(\xi)$ tends to zero as ξ tends to infinity

D $F(\xi) \in L^2(\mathbb{R})$

Options :

28860712921. 1

28860712922. 2

28860712923. 3

28860712924. 4

Question Number : 64 Question Id : 2886073238 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Which of the following is the recurrence relation for the power series solution of Legendre's equation $(1-x^2)y''-2xy'+p(p+1)y=0.$

A $\frac{a_{n+2}}{a_n}=\frac{n(n+1)+p(p+1)}{(n+1)(n+2)}$

B $\frac{a_{n+2}}{a_n}=\frac{n(n+1)-p(p+1)}{(n+1)(n+2)}$

C $\frac{a_{n+2}}{a_n}=\frac{n(n-1)-p(p-1)}{(n+1)(n+2)}$

D $\frac{a_{n+2}}{a_n}=\frac{n(n-1)+p(p-1)}{(n+1)(n+2)}$

Options :

28860712925. 1

28860712926. 2

28860712927. 3

28860712928. 4

Question Number : 65 Question Id : 2886073239 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The function $|x|^6$ defined on $[-\pi,\pi]$ and extended as a 2π periodic function is

A Holder continuous but not Lipschitz continuous

B Continuously differentiable

C Not Lipschitz continuous

D Lipschitz continuous but not continuously differentiable

Options :

28860712929. 1

28860712930. 2

28860712931. 3

28860712932. 4

Question Number : 66 Question Id : 2886073240 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The Fourier series of the function $|x|^5$ on $[-\pi,\pi]$ converges

A Absolutely but not uniformly

B Absolutely and uniformly

C Uniformly but not absolutely

D Conditionally and non-uniformly

Options :

28860712933. 1

28860712934. 2

28860712935. 3

28860712936. 4

Question Number : 67 Question Id : 2886073241 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

To conclude that a compact operator has an orthonormal basis of eigen-vectors which of the following hypothesis is sufficient?

A The operator is self-adjoint

B The operator is normal

C The operator is unitary

D The operator is positive

Options :

28860712937. 1

28860712938. 2

28860712939. 3

28860712940. 4

Question Number : 68 Question Id : 2886073242 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

Consider the function $f(x) = \pi - |x|$ on $(-\pi, \pi)$. The Fourier series for the function is

A $\pi + 4\sum_1^\infty \frac{\cos(2n-1)x)}{\pi(2n-1)^2}$

B $\pi + 2\sum_1^\infty \frac{\cos(2n-1)x)}{\pi(2n-1)^2}$

C $\frac{\pi}{2} + 4\sum_1^\infty \frac{\cos(2n-1)x)}{\pi(2n-1)^2}$

D $\frac{\pi}{2} + 2\sum_1^\infty \frac{\cos(2n-1)x)}{\pi(2n-1)^2}$

Options :

28860712941. 1

28860712942. 2

28860712943. 3

28860712944. 4

Question Number : 69 Question Id : 2886073243 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

By computing the Fourier series for x^2 or otherwise the value of

$\sum_1^{\infty} \frac{(-1)^{n+1}}{n^2}$ equals:

A $\frac{\pi^2}{12}$

B $\frac{\pi^2}{18}$

C $\frac{\pi^2}{24}$

D $\frac{\pi^2}{30}$

Options :

28860712945. 1

28860712946. 2

28860712947. 3

28860712948. 4

Question Number : 70 Question Id : 2886073244 Question Type : MCQ Option Shuffling : No
Correct Marks : 2 Wrong Marks : 0

The coefficient of $\cos 2nx$ in the Fourier series of $\cos ax$ (a is not an integer) is:

A $-\sin a\pi\ /(\pi(4n^2 - a^2))$

B $\sin a\pi\ /(\pi(4n^2 - a^2))$

C $2\,a\,(\sin a\pi)\ /(\pi(4n^2 - a^2))$

D $-2\,a\,(\sin a\pi)\ /(\pi(4n^2 - a^2))$

Options :

28860712949. 1

28860712950. 2

28860712951. 3

28860712952. 4