National Testing Agency

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Fourier Analysis and its Applications

Group Number :	1
Group Id :	89951411
Group Maximum Duration :	0
Group Minimum Duration :	120
Show Attended Group? :	No
Edit Attended Group? :	No
Break time :	0
Group Marks :	100
Is this Group for Examiner? :	No

Fourier Analysis and its Applications

Section Id :	89951411
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	70

Number of Questions to be attempted :	50
Section Marks :	100
Display Number Panel :	Yes
Group All Questions :	Yes
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	89951420
Question Shuffling Allowed :	Yes

Question Number : 1 Question Id : 899514871 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The constant term in the Fourier series expansion of $f(x) = |x|^3$ on the interval $[-\pi, \pi]$

equals:

A π^3

B $\pi^3/2$

C $\pi^3/3$

D $\pi^3/4$

Options :

8995143461. 1

8995143462. 2

8995143463. 3

8995143464. 4

Question Number : 2 Question Id : 899514872 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The value of $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ equals:

A $\pi^2/6$
B $\pi^2/8$
C $\pi^2/16$
D $\pi^2/18$

Options :

8995143465. 1
8995143466. 2
8995143467. 3
8995143468. 4

Question Number : 3 Question Id : 899514873 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Supply the constant b in the following formula:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}|f(x)|^2\,dx = |a_0|^2 + b\sum_1^{\infty}(|a_n|^2 + |b_n|^2)$$

A 1
B $\frac{1}{2}$
C π
D $\frac{\pi}{2}$

Options :

8995143469. 1
8995143470. 2
8995143471. 3
8995143472. 4

Question Number : 4 Question Id : 899514874 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The $n-th$ Legendre polynomial is $P_n(x)$. Then the value of $P_n(-1)$ equals:

A 1

B $(-1)^n$

C $\frac{1}{2^n n!}$

D $2^n n!$

Options :

8995143473. 1

8995143474. 2

8995143475. 3

8995143476. 4

Question Number : 5 Question Id : 899514875 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

What is the value of $\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n}$?

A e

B $\frac{1}{e}$

C 1

D $\sqrt{e}$

Options :

8995143477. 1

8995143478. 2

8995143479. 3

8995143480. 4

Question Number : 6 Question Id : 899514876 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is Fejer's kernel?

A $\frac{1}{2(n+1)}\left(\sin\frac{(n+1)\theta}{2}\right)^2\left(\sin\frac{\theta}{2}\right)^{-2}$

B $\frac{1}{(n+1)}\left(\sin\frac{(n+1)\theta}{2}\right)^2\left(\sin\frac{\theta}{2}\right)^{-2}$

C $\frac{1}{2\pi}\left(\sin\frac{(n+1)\theta}{2}\right)^2\left(\sin\frac{\theta}{2}\right)^{-2}$

D $\frac{1}{2(n+1)\pi}\left(\sin\frac{(n+1)\theta}{2}\right)^2\left(\sin\frac{\theta}{2}\right)^{-2}$

Options :

8995143481. 1

8995143482. 2

8995143483. 3

8995143484. 4

Question Number : 7 Question Id : 899514877 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The behavior of the Fejer kernel $K_n(t)$ near $t = 0$ is

A $\frac{n+1}{2}$

B $\frac{1}{n+1}$

C $\frac{n+1}{2\pi}$

D $\frac{1}{2\pi}$

Options :

8995143485. 1

8995143486. 2

8995143487. 3

8995143488. 4

Question Number : 8 Question Id : 899514878 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Weyl's equi-distribution theorem is a sharpening of

A Weierstrass's approximation theorem

B Kronecker's theorem

C Riemann Lebesgue Lemma

D Parseval's theorem

Options :

8995143489. 1

8995143490. 2

8995143491. 3

8995143492. 4

Question Number : 9 Question Id : 899514879 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is FALSE ?

A $\{n + m\pi : m, n \in \mathbb{Z}\}$ is dense in $\mathbb{R}$

B $\{n - [n\pi] : n \in \mathbb{N}\}$ is dense in $[0, 1]$

C $\{n + 22m/7 : m, n \in \mathbb{Z}\}$ is dense in $\mathbb{R}$

D $\{q - [qn] : n \in \mathbb{N}\}$ is dense in $[0, 1]$,where q is any irrational number

Options :

8995143493. 1

8995143494. 2

8995143495. 3

8995143496. 4

Question Number : 10 Question Id : 899514880 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following functions is not in the Schwartz's class?

A $1/(\cosh x)$

B $x/\sinh x$

C $\sinh x/\cosh x$

D $x^{100}/\cosh x$

Options :

8995143497. 1

8995143498. 2

8995143499. 3

8995143500. 4

Question Number : 11 Question Id : 899514881 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The function $f(x)$ equals 1 on the interval $[-1, 1]$ and is zero outside this interval.
The Fourier transform of $f(x)$ equals

A $\sin \xi/\xi$
B $2\sin \xi/\xi$
C $\cos \xi/\xi$
D $\sin \xi$

Options :

8995143501. 1
8995143502. 2
8995143503. 3
8995143504. 4

Question Number : 12 Question Id : 899514882 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Fourier transform of e^{-x^2} is $\sqrt{\pi}e^{-bx^2}$. Then the value of b equals

A ¼
B 4
C 1
D ½

Options :

8995143505. 1
8995143506. 2
8995143507. 3
8995143508. 4

Question Number : 13 Question Id : 899514883 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Supply the value of the constant c in the following formula (where $f(x)$ is in the Schwartz's class) $\hat{\hat{f}}(x) = cf(-x)$

A 2π
B π
C $\pi/2$
D 1

Options :

8995143509. 1
8995143510. 2
8995143511. 3
8995143512. 4

Question Number : 14 Question Id : 899514884 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is NOT the Fourier transform of a function in $L^2(\mathbb{R})$

A $|\xi|/(1+\cosh\xi)$
B $|\xi|/(1+|\xi|^2)$
C $\sin\xi\,/|\xi|$
D $\tanh\xi$

Options :

8995143513. 1
8995143514. 2
8995143515. 3
8995143516. 4

Question Number : 15 Question Id : 899514885 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Given that $J_0'(x) = J_p(x)$ (where $J_p(x)$ denotes the Bessel function of order p, the value of p equals:

A -1
B 1
C 2
C -3

Options :

8995143517. 1
8995143518. 2
8995143519. 3
8995143520. 4

Question Number : 16 Question Id : 899514886 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following differential equations is invariant under the Fourier transform?

A Airy's equation
B Hermite's equation
C Legendre's equation
D Tchebychev'

Options :

8995143521. 1
8995143522. 2
8995143523. 3
8995143524. 4

Question Number : 17 Question Id : 899514887 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

If $(f_n(x))$ is an orthonormal sequence of functions in $L^2[0,1]$ then which of the following is always true?

A The sequence converges in norm
B The sequence converges weakly
C The sequence converges pointwise
D The sequence converges uniformly

Options :

8995143525. 1
8995143526. 2
8995143527. 3
8995143528. 4

Question Number : 18 Question Id : 899514888 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Consider the function $f(x) = 1 - \cos x + \frac{1}{3}(\sin 3x - \cos 3x)$. The value of $\int_{-\pi}^{\pi} |f(x)|^2 dx$ equals:

A $\frac{20\pi}{9}$
B $\frac{23\pi}{9}$
C $\frac{26\pi}{9}$
D $\frac{29\pi}{9}$

Options :

8995143529. 1
8995143530. 2
8995143531. 3
8995143532. 4

Question Number : 19 Question Id : 899514889 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question

Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Given that $J_0(x) = 1/\pi \int_0^{\pi} \cos(x \sin\theta)\, d\theta$. Which of the following is true?

A $J_0(x) = \sum_0^{\infty} \frac{(-1)^n}{n!^2} \left(\frac{x}{2}\right)^{2n}$

B $J_0(x) = \sum_0^{\infty} \frac{1}{n!^2} \left(\frac{x}{2}\right)^{2n}$

C $J_0(x) = \sum_0^{\infty} \frac{1}{n!} \left(\frac{x}{2}\right)^{2n}$

D $J_0(x) = \sum_0^{\infty} \frac{(-1)^n}{n!} \left(\frac{x}{2}\right)^{2n}$

Options :
8995143533. 1
8995143534. 2
8995143535. 3
8995143536. 4

Question Number : 20 Question Id : 899514890 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Let X be the Banach space of continuous functions $f(x)$ on $[-\pi, \pi]$ such that $f(\pi) = f(-\pi)$ endowed with the supremum norm and S be the subset of X consisting of all the functions in X whose Fourier series converges pointwise everywhere. Then

A S equals X.

B S is not equal to X but S is a countable intersection of dense open sets in X.

C $X - S$ is dense in X

D $X - S$ is a countable union of closed sets each having empty interior in X

Options :

8995143537. 1
8995143538. 2
8995143539. 3
8995143540. 4

Question Number : 21 Question Id : 899514891 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following represents the Poisson Kernel?

A $\frac{1}{2\pi}\frac{1-r^2}{1+r^2-2r\cos\theta}$

B $\frac{1}{\pi}\frac{1-r^2}{1+r^2-2r\cos\theta}$

C $\frac{1}{2\pi}\frac{1+r^2}{1+r^2-2r\cos\theta}$

D $\frac{1}{\pi}\frac{1+r^2}{1+r^2-2\cos\theta}$

Options :

8995143541. 1
8995143542. 2
8995143543. 3
8995143544. 4

Question Number : 22 Question Id : 899514892 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The Lebesgue constants are the numbers $L_n = \int_0^{\pi} |D_n(t)| dt$ where $D_n(t)$ is the Dirichlet kernel. Which of the following is true?

A $\sum_0^{\infty} L_n$ converges.

B $L_n \sim c \log n$ for some constant c and for large n

C $L_n \to 0$ as $n \to \infty$

D $L_n \sim c\sqrt{n} \log n$ for large n

Options :

8995143545. 1

8995143546. 2

8995143547. 3

8995143548. 4

Question Number : 23 Question Id : 899514893 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Consider the solution $u(x, y)$ of $\Delta u = 0$ on $\{(x, y): x^2 + y^2 < 1\}$ and $u(\cos\theta, \sin\theta) = |\sin\theta|$. Then the value of $u(0, 0)$ equals:

A $\frac{\pi}{2}$

B $1/\pi$

C $2/\pi$

D π

Options :

8995143549. 1

8995143550. 2

8995143551. 3

8995143552. 4

Question Number : 24 Question Id : 899514894 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Consider the functions x^n on $L^2(-1,1)(n=0,1,2,3,\ldots)$. When these are subjected to the Gram-Schmidt process the resulting functions are $f_n(x)$. Then which of the following is true?

A The $f_n(x)$ are the Legendre polynomials
B The $f_n(x)$ are the Tchebycheff's polynomials
C The $f_n(x)$ are the Hermite polynomials
D The $f_n(x)$ are the Laguerre polynomials

Options :

8995143553. 1
8995143554. 2
8995143555. 3
8995143556. 4

Question Number : 25 Question Id : 899514895 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

A solution to the wave equation $u_{tt} - c^2\Delta u = 0$ are sought in the form $u(x,\ y,\ t) = e^{ikt}\ v(x,y)$. Then $v(x,\ y)$ satisfies which of the following equations?

A $\Delta v - (k^2/c^2)v = 0.$
B $\Delta v + (k^2/c^2)v = 0.$
C $\Delta v + k^2 v = 0.$
D $c^2\Delta v + v = 0.$

Options :

8995143557. 1
8995143558. 2
8995143559. 3
8995143560. 4

Question Number : 26 Question Id : 899514896 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Coefficients a_n, b_n in the Fourier series expansion of a 2π periodic function behave like n^{-2-c} where c is a positive number. Then which of the following is true?

A The sun of the series is continuous but not uniformly continuous.

B The sun of the series is uniformly continuous but not absolutely continuous.

C The sun of the series is absolutely continuous but not continuously differentiable

D The sun of the series is continuously differentiable.

Options :

8995143561. 1

8995143562. 2

8995143563. 3

8995143564. 4

Question Number : 27 Question Id : 899514897 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following functions is in the Schwartz's class? Here $\Gamma(\xi)$ is the Gamma function and its reciprocal is assigned value 0 at the 0 and the negative integers.

A $\sin\xi\ /\xi$

B $\sin\xi/(1+\ \xi^2)$

C $\sin\xi/(\Gamma(1+\xi^2))$

D $(\sin\xi)\exp(-|\xi|)$

Options :

8995143565. 1

8995143566. 2

8995143567. 3

8995143568. 4

Question Number : 28 Question Id : 899514898 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is true about the Fourier transform as an operator from $L^2(\mathbb{R})$ to itself

A It is unitary

B It is selfadjoint

C It is a compact operator

D It is an unbounded operator.

Options :

8995143569. 1

8995143570. 2

8995143571. 3

8995143572. 4

Question Number : 29 Question Id : 899514899 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The value of $\int_0^{2\pi} \frac{d\theta}{(1+r^2-2r\cos\theta)}$ equals:

A $2\pi/(1+r^2)$

B $2\pi/(1-r^2)$

C $\pi/(1+r^2)$

D $\pi/(1-r^2$

Options :

8995143573. 1

8995143574. 2

8995143575. 3

8995143576. 4

Question Number : 30 Question Id : 899514900 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Apply the Parseval formula to the function $f(x) = x/|x|, f(0) = 0$ on $[-\pi, \pi]$.
Which of the following do you deduce as the value of
$1 + \frac{1}{3^2} + \frac{1}{5^2} \ldots$

A $\pi^2/3$
B $\pi^2/4$
C $\pi^2/6$
D $\pi^2/8$

Options :

8995143577. 1
8995143578. 2
8995143579. 3
8995143580. 4

Question Number : 31 Question Id : 899514901 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

For a compact operator on in $L^2[0, 1]$ which of the following is false?

A The spectrum is non-empty
B The spectrum can be uncountable
C The spectrum is at most countable
D The spectrum in certain cases can be $\{0\}$

Options :

8995143581. 1
8995143582. 2
8995143583. 3
8995143584. 4

Question Number : 32 Question Id : 899514902 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The polynomials $\exp(\frac{x}{2})\left(\frac{d}{dx}\right)^n (x^n \exp -(\frac{x}{2}))$ satisfies which of the following equations?

A Legendre equation
B Hermite's equation
C Laguerre equation
D Tchebychev's equation

Options :

8995143585. 1
8995143586. 2
8995143587. 3
8995143588. 4

Question Number : 33 Question Id : 899514903 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Given that $f(x) = \exp(-|x|)$ what is the value of $\hat{f}(0)$? Here $\hat{f}$ is the Fourier transform of f

A 1
B 2
C π
D 2π

Options :

8995143589. 1
8995143590. 2
8995143591. 3
8995143592. 4

Question Number : 34 Question Id : 899514904 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Given that $x^6 = c_6P_6(x) + c_5P_5(x) + \cdots + c_0P_0(x)$ where $P_n(x)$ is the $n-th$ Legendre polynomial, the value of c_6 equals:

A $16/191$
B $16/211$
C $16/231$
D $16/251$

Options :

8995143593. 1
8995143594. 2
8995143595. 3
8995143596. 4

Question Number : 35 Question Id : 899514905 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

What is the value of $2\int_0^1 P_5(x)dx$? Here $P_n(x)$ is the n-th Legendre polynomial.

A $1/8$
B $1/16$
C $1/32$
D 0

Options :

8995143597. 1
8995143598. 2
8995143599. 3
8995143600. 4

Question Number : 36 Question Id : 899514906 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The $n-th$ Legendre polynomial is $P_n(x)$. Then the value of $P'_{11}(-1)$ equals

A 55
B 66
C 50
D 60

Options :

8995143601. 1
8995143602. 2
8995143603. 3
8995143604. 4

Question Number : 37 Question Id : 899514907 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following statement about the Volterra operator $T : L^2[0,1] \to L^2[0,1]$ is true?

A It has no eigen-values
B It is selfadjoint
C It is a bounded non-compact operator
D 0 is in the spectrum of T

Options :

8995143605. 1
8995143606. 2
8995143607. 3
8995143608. 4

Question Number : 38 Question Id : 899514908 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is a trigonometric polynomial on $[-\pi, \pi]$

A $\sin(\frac{x}{3})$

B $\sin^3 x$

C $\sin^{1/3} x$

D $\sin \sqrt{3}x$

Options :

8995143609. 1

8995143610. 2

8995143611. 3

8995143612. 4

Question Number : 39 Question Id : 899514909 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

If $f(x) = 1/(1 + x^2)$ then what is the value of $f * f(0)$? Note $f * g$ denotes the convolution of f and g

A 2π

B π

C $\frac{\pi}{2}$

D $\frac{\pi}{4}$

Options :

8995143613. 1

8995143614. 2

8995143615. 3

8995143616. 4

Question Number : 40 Question Id : 899514910 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The function $f(x)$ equals 1 on the interval $[-1, 1]$ and is zero outside this interval. What is the Fourier transform of $f * f$. Note $f * g$ denotes the convolution of f and g

A $\frac{\sin^2 \xi}{\xi}$

B $\frac{2\sin^2 \xi}{\xi}$

C $\frac{2\sin^2 \xi}{\xi^2}$

D $\frac{4\sin^2 \xi}{\xi^2}$

Options :

8995143617. 1

8995143618. 2

8995143619. 3

8995143620. 4

Question Number : 41 Question Id : 899514911 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

A sequence $(v_n(x))$ of functions in $L^2[0, 1]$ converges to $v(x)$ weakly. Which of the following is true?

A The sequence is norm convergent

B A subsequence converges pointwise

C The sequence converges pointwise

D The sequence is norm-bounded

Options :

8995143621. 1

8995143622. 2
8995143623. 3
8995143624. 4

Question Number : 42 Question Id : 899514912 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Which of the following is an even function?

A $J_0'(x)$
B $P_4'(x)$
C Fourier transform of an even function
D Fourier transform of an odd function

Options :
8995143625. 1
8995143626. 2
8995143627. 3
8995143628. 4

Question Number : 43 Question Id : 899514913 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Which of the following is Luzin's theorem?

A $C[0,1]$ is dense in $L^2[0,1]$ with respect to L^2 norm
B Polynomials are dense in $L^2[0,1]$ with respect to L^2 norm
C Polynomials are dense in $C[0,1]$ with respect to supremum norm
D $C[0,1]$ is dense in $L^\infty[0,1]$ with respect to L^∞ norm

Options :
8995143629. 1
8995143630. 2
8995143631. 3
8995143632. 4

Question Number : 44 Question Id : 899514914 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Fourier transform of a radial function is

A Always radial and given by a sine transform

B Always radial and given by a Bessel transform in two dimensions

C Always radial and given by a Bessel transform in three dimensions

D Always radial and given by a cosine transform

Options :

8995143633. 1

8995143634. 2

8995143635. 3

8995143636. 4

Question Number : 45 Question Id : 899514915 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

For the heat equation $u_t - u_{xx} = 0$ with initial condition $u(x,0) = f(x)$. Assume that $f(x)$ is positive outside $(-1,1)$ and zero in $(-1,1)$. Then

A $u(x,t)$ is positive for all $x \in \mathbb{R}$ and for all positive t

B $u(x,t)$ is positive only for x outside $(-1,1)$ and for all positive t

C $u(x,t)$ is positive only for x outside $(-2,2)$ and for all positive t

D $u(x,t)$ is positive only for x outside $(0,2)$ and for all positive t

Options :

8995143637. 1

8995143638. 2

8995143639. 3

8995143640. 4

Question Number : 46 Question Id : 899514916 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question

Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

For the wave equation $u_{tt} - u_{xx} = 0$ with initial conditions in the Schwartz class, assume that $E(t)$ is the total energy and $K(t)$ is the kinetic energy. Then, which of the following is false?

A $E(t)$ is constant in time
B $K(t)$ is not constant in time
C $K(t)$ approaches $E(0)$ as time goes to infinity
D $K(t)$ approaches $E(0)/2$ as time goes to infinity

Options :
8995143641. 1
8995143642. 2
8995143643. 3
8995143644. 4

Question Number : 47 Question Id : 899514917 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

The sum $1 + 2\cos 2x + 2\cos 4x + \cdots + 2\cos 2nx$ equals:

A $\dfrac{\sin(nx + x)}{\sin\ x}$

B $\dfrac{\sin(2nx + x)}{\sin\ x}$

C $\dfrac{\sin(2nx)}{\sin\ x}$

D $\dfrac{\sin(nx)}{\sin\ x}$

Options :
8995143645. 1

8995143646. 2
8995143647. 3
8995143648. 4

Question Number : 48 Question Id : 899514918 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Which of the following is true about the $n-th$ Legendre polynomial?

A It has n distinct roots in $(-1,1)$
B It has $+1$ as a simple root
C It has a double root at the origin
D It has a double root at both 1 and -1

Options :
8995143649. 1
8995143650. 2
8995143651. 3
8995143652. 4

Question Number : 49 Question Id : 899514919 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Suppose T_n is a sequence of continuous linear maps from a Banach space X into $\mathbb{R}$ such that for each $x \in X$ the sequence $T_n(x)$ is bounded then which of the following is false?

A $||T_n||$ is bounded
B $\left|\left|T_n(x)\right|\right|$ has a bound independent of x for all $\left|\left|x\right|\right| \leq 1$
C $||T_n||$ converges
D $||T_n||$ has a convergent subsequence.

Options :
8995143653. 1
8995143654. 2

8995143655. 3
8995143656. 4

Question Number : 50 Question Id : 899514920 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

The Banach Steinhaus's theorem implies

A Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series converges uniformly
B Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series converges pointwise
C Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series diverges at a specific point.
D Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series diverges everywhere.

Options :
8995143657. 1
8995143658. 2
8995143659. 3
8995143660. 4

Question Number : 51 Question Id : 899514921 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

Which of the following is not a trigonometric polynomial on $[-\pi, \pi]$

A $\cos 5x$
B $\cos^{1/5} x$
C $\cos^5 x$
D $5 \cos x$

Options :
8995143661. 1
8995143662. 2
8995143663. 3

8995143664. 4

Question Number : 52 Question Id : 899514922 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The Fourier transform of the function $f(x) = e^{-|x|}$ equals:

A $2/(1+\xi^2)$

B $\xi/(1+\xi^2)$

C $1/(1+\xi^2)$

D $\pi/(1+\xi^2)$

Options :

8995143665. 1

8995143666. 2

8995143667. 3

8995143668. 4

Question Number : 53 Question Id : 899514923 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following about the Bessel's function $J_p(z)$ is false?

A It is an entire function of z for positive integer values of p

B It is not an entire function of z for non-integer values of p

C It is an entire function of z for negative integer values of p

D It is not an entire function of z for any value of p

Options :

8995143669. 1

8995143670. 2

8995143671. 3

8995143672. 4

Question Number : 54 Question Id : 899514924 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The Weierstrass's approximation theorem implies

A The separability of $C[0,1]$

B The separability of $L^{\infty}[0,1]$

C That $C[0,1]$ is not separable

D That $L^{1}[0,1]$ is not separable.

Options :

8995143673. 1

8995143674. 2

8995143675. 3

8995143676. 4

Question Number : 55 Question Id : 899514925 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is the Bessel's equation of order p?

A $x^2y'' + xy' + (x^2 + p^2)\,\mathrm{y} = 0$

B $x^2y'' - xy' + (x^2 - p^2)\,\mathrm{y} = 0$

C $x^2y'' - xy' + (x^2 + p^2)\,\mathrm{y} = 0$

D $x^2y'' + xy' + (x^2 - p^2)\,\mathrm{y} = 0$

Options :

8995143677. 1

8995143678. 2

8995143679. 3

8995143680. 4

Question Number : 56 Question Id : 899514926 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

For vectors v and $w$in a Hilbert space, given that $||v|| = 4$, $||w|| = 4$ and $||w - v|| = 3$.
Then what is the value of $||w + v||$?

A $\sqrt{47}$

B $\sqrt{51}$

C $\sqrt{53}$

D $\sqrt{55}$

Options :

8995143681. 1

8995143682. 2

8995143683. 3

8995143684. 4

Question Number : 57 Question Id : 899514927 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

If $F(\xi)$ is the Fourier transform of $f(x)$ then the Fourier transform of $f''(x)$ equals:

A $-i\xi^2 F(\xi)$

B $i\xi^2 F(\xi)$

C $-\xi^2 F(\xi)$

D $\xi^2 F(\xi)$

Options :

8995143685. 1

8995143686. 2

8995143687. 3

8995143688. 4

Question Number : 58 Question Id : 899514928 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

If $F(\xi)$ is the Fourier transform of $f(x)$ then the Fourier transform of $x^2 f(x)$ equals:

A $-iF''(\xi)$

B $iF''(\xi)$

C $-F''(\xi)$

D $F''(\xi)$

Options :

8995143689. 1

8995143690. 2

8995143691. 3

8995143692. 4

Question Number : 59 Question Id : 899514929 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is true?

A $C[-\pi, \pi]$ with respect to sup norm has countable Hamel basis

B $C[-\pi, \pi]$ with respect to sup norm has uncountable Hamel basis

C The set of all polynomials has uncountable Hamel basis

D $L^{\infty}[-\pi, \pi]$ with respect to L^{∞} norm has countable Hamel basis

Options :

8995143693. 1

8995143694. 2

8995143695. 3

8995143696. 4

Question Number : 60 Question Id : 899514930 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Let $y(x)$ be a solution of the differential equation $y'' + xy = 0$. Then which of the following is true?

A The zeros of $y(x)$ form a finite set

B The zeros of $y(x)$ are equally spaced

C The zeros (z_n) of $y(x)$ are countable discrete increasing sequence and $|z_n - z_{n-1}|$ tends to zero as n tends to infinity.

D The zeros of $y(x)$ have a finite limit point

Options :

8995143697. 1

8995143698. 2

8995143699. 3

8995143700. 4

Question Number : 61 Question Id : 899514931 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is an essential hypothesis on a metric space X in order to apply the Baire Category theorem?

A X is complete

B X is locally connected

C X is connected

D X is separable

Options :

8995143701. 1

8995143702. 2

8995143703. 3

8995143704. 4

Question Number : 62 Question Id : 899514932 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Given that F is the Fourier transform of a Schwartz function f

Supply the value of c in the following formula

$$\int_{-\infty}^{\infty} |f(x)|^2 \, dx = c \int_{-\infty}^{\infty} |F(\xi)|^2 \, d\xi$$

A 2π

B $\sqrt{2\pi}$

C $(2\pi)^{-1}$

D $\left(\sqrt{2\pi}\right)^{-1}$

Options :

8995143705. 1

8995143706. 2

8995143707. 3

8995143708. 4

Question Number : 63 Question Id : 899514933 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Given that $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ which of the following always hold? Here $F(\xi)$ is the Fourier transform of f

A $F(\xi) \in L^1(\mathbb{R})$

B $F(\xi)$ is uniformly continuous

C $F \notin L^2(\mathbb{R})$

D $F(\xi)$ is absolutely continuous

Options :

8995143709. 1

8995143710. 2
8995143711. 3
8995143712. 4

Question Number : 64 Question Id : 899514934 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Which of the following is the Legendre's differential equation?

A $(1-x^2)y'' + 2xy' + p(p+1)y = 0$

B $(1-x^2)y'' - 2xy' + p(p+1)y = 0$

C $(1+x^2)y'' + 2xy' + p(p+1)y = 0$

D $(1+x^2)y'' - 2xy' + p(p+1)y = 0$

Options :

8995143713. 1
8995143714. 2
8995143715. 3
8995143716. 4

Question Number : 65 Question Id : 899514935 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The function $|x|^2$ defined on $[-\pi, \pi]$ and extended as a 2π periodic function is

A Holder continuous but not Lipschitz continuous

B Continuously differentiable

C Not Lipschitz continuous

D Lipschitz continuous but not continuously differentiable

Options :

8995143717. 1

8995143718. 2
8995143719. 3
8995143720. 4

Question Number : 66 Question Id : 899514936 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

The Fourier series of the function $|x|^3$ on $[-\pi, \pi]$ converges

A Conditionally and non-uniformly

B Absolutely and uniformly

C Absolutely but not uniformly

D Uniformly but not absolutely

Options :
8995143721. 1
8995143722. 2
8995143723. 3
8995143724. 4

Question Number : 67 Question Id : 899514937 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical
Correct Marks : 2 Wrong Marks : 0

To conclude that a compact operator has an orthonormal basis of eigen-vectors which of the following hypothesis is sufficient?

A The operator is self-adjoint

B The operator is normal

C The operator is unitary

D The operator is positive

Options :

8995143725. 1
8995143726. 2
8995143727. 3
8995143728. 4

Question Number : 68 Question Id : 899514938 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Consider the function $f(x) = \pi - x$ on $(0, \pi)$ and $f(x) = -(\pi + x)$ on $(-\pi, 0)$. The Fourier series for the function is

A $\sum_{1}^{\infty} \frac{\sin(nx)}{2n}$

B $\sum_{1}^{\infty} \frac{\sin(nx)}{n}$

C $2\sum_{1}^{\infty} \frac{\sin(nx)}{n}$

D $\sum_{1}^{\infty} \frac{\sin(2nx)}{n}$

Options :

8995143729. 1
8995143730. 2
8995143731. 3
8995143732. 4

Question Number : 69 Question Id : 899514939 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

Parseval formula applied to $f(x) = x^2$ give the value of

$1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$ as

A $\frac{\pi^4}{90}$

B $\frac{\pi^4}{96}$

C $\frac{\pi^4}{98}$

D $\frac{\pi^4}{112}$

Options :

8995143733. 1

8995143734. 2

8995143735. 3

8995143736. 4

Question Number : 70 Question Id : 899514940 Question Type : MCQ Option Shuffling : No Display Question Number : Yes Is Question Mandatory : No Single Line Question Option : No Option Orientation : Vertical

Correct Marks : 2 Wrong Marks : 0

The coefficient of $\cos(2n-1)x$ in the Fourier series of $\cos ax$ (a is not an integer) is:

A $\mathrm{a}\,(\sin a\pi)\,/(\pi((2n-1)^2 - a^2))$

B $-a/(\pi((2n-1)^2 - a^2))$

C $-2\,\mathrm{a}\,(\sin a\pi)\,/(\pi((2n-1)^2 - a^2))$

D $2\,\mathrm{a}\,(\sin a\pi)\,/(\pi((2n-1)^2 - a^2))$

Options :

8995143737. 1

8995143738. 2

8995143739. 3

8995143740. 4