National Testing Agency

Question Paper Name :	Topology 29 Sep 20 Shift 1
Subject Name :	Topology
Creation Date :	2020-09-29 13:08:32
Duration :	180
Number of Questions :	34
Total Marks :	100
Display Marks:	Yes

Topology

Group Number :	1
Group Id :	899514107
Group Maximum Duration :	0
Group Minimum Duration :	120
Show Attended Group? :	No
Edit Attended Group? :	No
Break time :	0
Group Marks :	100
Is this Group for Examiner? :	No

Topology-1

Section Id :	899514133
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20
Number of Questions to be attempted :	20

Section Marks : 20
Mark As Answered Required? : Yes
Sub-Section Number : 1
Sub-Section Id : 899514165
Question Shuffling Allowed : Yes

Question Number : 1 Question Id : 8995149443 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which of the following statements is correct?

1. $\emptyset \in \{0\}$
2. $\emptyset \subset \{0\}$
3. $x \in \{\{x\}\}$
4. $\{0\} = \emptyset$

Options :

89951437178. 1
89951437179. 2
89951437180. 3
89951437181. 4

Question Number : 2 Question Id : 8995149444 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

If $f:\mathbb{R} \to \mathbb{R}$is injective then any line parallel to X axis

1. cuts graph of f exactly at one point
2. cuts graph of f atmost at one point
3. always cuts graph of f at atleast two points
4. never cuts graph of f

Options :

89951437182. 1
89951437183. 2
89951437184. 3
89951437185. 4

Question Number : 3 Question Id : 8995149445 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which one of the following sets is countably infinite?

1. set of all even integers
2. $(0,1]\setminus\{\frac{1}{n} : n \in \mathbb{Z}\}$
3. $\{n \in \mathbb{Z} : n^2 < 5\}$
4. none of these

Options :

89951437186. 1

89951437187. 2

89951437188. 3

89951437189. 4

Question Number : 4 Question Id : 8995149446 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Let d_1 and d_2 be metrics on a non-empty set X. For $x, y \in X$ let $d(x,y) = min\,\{d_1(x,y), d_2(x,y)\}$ and $d'(x,y) = max\,\{d_1(x,y), d_2(x,y)\}$. Then

1. Both d, d' are metrics on X
2. d is a metric on X, d' is not
3. d' is a metric on X, d is not
4. None of the above

Options :

89951437190. 1

89951437191. 2

89951437192. 3

89951437193. 4

Question Number : 5 Question Id : 8995149447 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Let $a \in \mathbb{R}$ and let $A = \mathbb{R} - \{a\}$. Then $A^0 =$

1. $\mathbb{R}$
2. $\{a\}$
3. $\mathbb{R}\backslash\{a\}$
4. None of these

Options :

89951437194. 1
89951437195. 2
89951437196. 3
89951437197. 4

Question Number : 6 Question Id : 8995149448 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

Let $A = (0,1) \subseteq \mathbb{R}$. Then

1. Each point of A is a limit point of A

2.0 and 1 are limit points of A

3. The set of limit points of A is $[0,1]$

4. Nothing can be said about the limit points of A, because the underlying metric is not specified

Options :

89951437198. 1
89951437199. 2
89951437200. 3
89951437201. 4

Question Number : 7 Question Id : 8995149449 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

Consider the discrete metric space X. Then

1. Each subset of X is open
2. Each subset of X is closed
3. Each subset of X is open as well as closed
4. Each of the above statements is true

Options :

89951437202. 1

89951437203. 2

89951437204. 3

89951437205. 4

Question Number : 8 Question Id : 8995149450 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which of the following sequences are convergent in the corresponding metric spaces?

1. $\left(\left(1+\frac{1}{n}\right)^{n}\right)$ in $\mathbb{Q}$ with usual metric.
2. $(1,0,1,0,\cdots)$ in $\mathbb{R}$ with usual metric
3. (f_n) ,where $f_n(x) = x^n$, $x \in [0,1]$ in $C[0,1]$ with $||.||_1$ metric
4. $((-1)^n)$ in $\mathbb{R}$ with usual metric

Options :

89951437206. 1

89951437207. 2

89951437208. 3

89951437209. 4

Question Number : 9 Question Id : 8995149451 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The *Bolzanno-Weierstrass' Theorem* states that:

1.Every Cauchy sequence in a metric space $(X, d)$is convergent

2.Every bounded sequence in $\mathbb{R}^n$ is convergent.

3.Every convergent sequence in a metric space (X, d) is convergent

4.Every bounded sequence in $\mathbb{R}^n$ with usual metric has a convergent subsequence

Options :

89951437210. 1
89951437211. 2
89951437212. 3
89951437213. 4

Question Number : 10 Question Id : 8995149452 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A uniform continuous function:

1.maps Cauchy sequences to Cauchy sequences

2. is bijective

3. may not map convergent sequences to convergent sequences

4. need not be continuous

Options :

89951437214. 1
89951437215. 2
89951437216. 3
89951437217. 4

Question Number : 11 Question Id : 8995149453 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Consider the $\mathbb{R}$ with usual metric. If $Fn = (0, \frac{1}{n})$, $\forall n \in \mathbb{N}$ then

1.$Fn's$ are closed in $\mathbb{R}$

2. $\text{diam}(Fn) \nrightarrow 0$

3. $Fn's$ are not a decreasing sequence of sets

4.$\bigcap_0^\infty F_n = \emptyset$

Options :

89951437218. 1

89951437219. 2

89951437220. 3

89951437221. 4

Question Number : 12 Question Id : 8995149454 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

In $(\mathbb{R}^2, d)$ where d is Euclidean distance, the following set is not connected.

1. $\mathbb{R}^2 \setminus \mathbb{Q} \times \mathbb{Q}$

2. $\mathbb{R}^2 \setminus \{(x, y): y = 0\}$

3.$\mathbb{R}^2 \setminus \{(0,0)\}$

4.None of the above

Options :

89951437222. 1

89951437223. 2

89951437224. 3

89951437225. 4

Question Number : 13 Question Id : 8995149455 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which of the following statements is false?

1. A compact subset of a metric space is closed and bounded
2. A finite subset of a metric space is compact
3. A closed subset of a compact set in a metric space is compact
4. A closed and bounded subset of a metric space is compact

Options :

89951437226. 1

89951437227. 2

89951437228. 3

89951437229. 4

Question Number : 14 Question Id : 8995149456 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which of the following is dense in $\mathbb{R}^2$ with respect to the usual topology?

1. $\{(x,y) \in \mathbb{R}^2 \mid x \in \mathbb{N}\}$.
2. $\{(x,y) \in \mathbb{R}^2 \mid x + y \ \ is\ an\ integer\}$
3. $\{(x,y) \in \mathbb{R}^2 \mid x + y^2 = 5\}$
4. $\{(x,y) \in \mathbb{R}^2 \mid xy \neq 0\}$.

Options :

89951437230. 1

89951437231. 2

89951437232. 3

89951437233. 4

Question Number : 15 Question Id : 8995149457 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Let Y be a subspace of X. If U is open in Y and Y is open is X
1.then U is closed in X.
2. then U is open in X.
3. then U is null set in X.
4.None of these.

Options :
89951437234. 1
89951437235. 2
89951437236. 3
89951437237. 4

Question Number : 16 Question Id : 8995149458 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Every compact subset of a Hausdorff space is
1. Closed set.
2. Open set.
3. Null set.
4. None of these.

Options :
89951437238. 1
89951437239. 2
89951437240. 3
89951437241. 4

Question Number : 17 Question Id : 8995149459 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A closed subset of a Baire space X is
1. always a Baire space.
2. never a Baire space.
3. a Baire space only when X is empty.
4. not necessarily a Baire space.

Options :
89951437242. 1
89951437243. 2
89951437244. 3
89951437245. 4

Question Number : 18 Question Id : 8995149460 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Let $f : X \to Y$ be bijective continuous function. If X is compact and Y is Hausdorff then
1. f is automorphism.
2. f is isomorphism.
3. f is Homeomorphism.
4. f^{-1} is not necessarily continuous.

Options :
89951437246. 1
89951437247. 2
89951437248. 3
89951437249. 4

Question Number : 19 Question Id : 8995149461 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

If X is a compact metric space then every open cover of X
1. has a Lebesgue number.
2. not necessarily has a Lebesgue number.
3. has a Lebesgue number only if X is an uncountable set.
4. None of these.

Options :
89951437250. 1
89951437251. 2
89951437252. 3
89951437253. 4

Question Number : 20 Question Id : 8995149462 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

The real line $\mathbb{R}$ with the finite complement topology.
1. closed.
2. compact.
3. not compact.
4. None of these

Options :
89951437254. 1
89951437255. 2
89951437256. 3
89951437257. 4

Topology-2

Section Id :	899514134
Section Number :	2
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	7
Number of Questions to be attempted :	5
Section Marks :	30
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	899514166
Question Shuffling Allowed :	No

Question Number : 21 Question Id : 8995149463 Question Type : SUBJECTIVE
Correct Marks : 6

Suppose (X, d) is a metric space and suppose $A \subseteq X$. Then prove the following:

(i) $A^o \subseteq A$

(ii) A^o is open

(iii) A is open if and only if $A = A^o$

Question Number : 22 Question Id : 8995149464 Question Type : SUBJECTIVE
Correct Marks : 6

Define the limit point of the set. Let$A = \mathbb{N} \subseteq \mathbb{R}$ under the Euclidean metric. Then show that the set of limit point of A is $\emptyset$.

Question Number : 23 Question Id : 8995149465 Question Type : SUBJECTIVE
Correct Marks : 6

Define a metric space. Let (X, d) be a metric space. Prove that $d': X \times X \to \mathbb{R}$ is a metric on X, where d' is defined as $d'(x, y) = \frac{d(x,y)}{1+d(x,y)}$ $\forall\, x, y \in X$. Also show that d' is bounded.

Question Number : 24 Question Id : 8995149466 Question Type : SUBJECTIVE
Correct Marks : 6

Define uniform continuity of function. Also prove that if f is continuous function on closed and bounded subset F of $\mathbb{R}$. Then f is uniformly continuous on F.

Question Number : 25 Question Id : 8995149467 Question Type : SUBJECTIVE
Correct Marks : 6

Define second countable space. Show that every second countable topological space is separable.

Question Number : 26 Question Id : 8995149468 Question Type : SUBJECTIVE
Correct Marks : 6

What is Heridiatry Property. Prove that discreteness is Heridiatry property.

Question Number : 27 Question Id : 8995149469 Question Type : SUBJECTIVE
Correct Marks : 6

State and prove Urysohn's lemma of metric space.

Topology-3

Section Id :	899514135
Section Number :	3
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	7
Number of Questions to be attempted :	5
Section Marks :	50
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	899514167
Question Shuffling Allowed :	No

Question Number : 28 Question Id : 8995149470 Question Type : SUBJECTIVE
Correct Marks : 10

State and prove sequential continuity theorem.

Question Number : 29 Question Id : 8995149471 Question Type : SUBJECTIVE
Correct Marks : 10

Show that the following are equivalent on the space X:

(i) X is connected.
(ii) The only subsets of X which are open and closed are X and $\emptyset$.
(iii) X cannot be express as union of two disjoint non-empty open sets.
(iv) There is no onto continuous function from X to a discrete space which contains more than one point.

Question Number : 30 Question Id : 8995149472 Question Type : SUBJECTIVE
Correct Marks : 10

State and prove Cantor's intersection theorem.

Question Number : 31 Question Id : 8995149473 Question Type : SUBJECTIVE
Correct Marks : 10

Show that every complete metric space is a Baire space.

Question Number : 32 Question Id : 8995149474 Question Type : SUBJECTIVE
Correct Marks : 10

State and prove Tube Lemma. Also, prove the results which will be use to prove this lemma.

Question Number : 33 Question Id : 8995149475 Question Type : SUBJECTIVE
Correct Marks : 10

Does every metric space is topological space? What about the converse?

Question Number : 34 Question Id : 8995149476 Question Type : SUBJECTIVE
Correct Marks : 10

Let X, Y be topological spaces; let $p : X \to Y$ be a quotient map; let A be saturated subset of X with respect to map p; let q be a map from A to $p(A)$ such that $q(a) = p(a)$ for all $a \in A$. Then prove the following.

(i) If A is either open or closed in X then q is quotient map.

(ii) If p is either open map or closed map, then q is a quotient map.