National Testing Agency

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Partial Differential Equations

Group Number :	1
Group Id :	899514130
Group Maximum Duration :	0
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Show Attended Group? :	No
Edit Attended Group? :	No
Break time :	0
Group Marks :	100
Is this Group for Examiner? :	No

Partial Differential Equations-1

Section Id :	899514169
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	1
Number of Questions to be attempted :	1

Section Marks : 20
Mark As Answered Required? : Yes
Sub-Section Number : 1
Sub-Section Id : 899514210
Question Shuffling Allowed : Yes

Question Id : 89951411277 Question Type : COMPREHENSION Sub Question Shuffling Allowed : Yes Group Comprehension Questions : No

Question Numbers : (1 to 20)

Question Label : Comprehension

Note: The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Sub questions

Question Number : 1 Question Id : 89951411278 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

$xp + yq = z$ is an example of

(a) a semi-linear equation

(b) a quasi-linear equation

(c) a non-linear equation

(d) a linear equation

Options :

89951444282. 1
89951444283. 2
89951444284. 3
89951444285. 4

Question Number : 2 Question Id : 89951411279 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

The partial differential equation formed by eliminating arbitrary function f from the PDE $z = x + y + f(xy)$ is given by

(a) $px + qy = x + y$

(b) $py + qx = x - y$

(c) $px - qy = x - y$

(d) $py - qx = x + y$

Options :

89951444286. 1

89951444287. 2

89951444288. 3

89951444289. 4

Question Number : 3 Question Id : 89951411280 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The solution of the PDE $(2D^2 - 5DD' + 2D'^2)z = 0$ is given by

(a) $z = \phi_1(y + 2x) + \phi_2(x + 2y)$

(b) $z = \phi_1(y - 2x) + \phi_2(y + 2x)$

(c) $z = \phi_1(x + 2y) + \phi_2(x - 2y)$

(d) $z = \phi_1(y - 2x) + \phi_2(x - 2y)$

Options :

89951444290. 1

89951444291. 2

89951444292. 3

89951444293. 4

Question Number : 4 Question Id : 89951411281 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The partial differential equation formed by eliminating arbitrary constants from the PDE $ax^2 + by^2 + z^2 = 1$ is given by

(a) $z(px + qy) = z^2 + 1$

(b) $z(px + qy) = z^2 - 1$

(c) $z(px - qy) = z^2 - 1$

(d) none of the above

Options :

89951444294. 1

89951444295. 2

89951444296. 3

89951444297. 4

Question Number : 5 Question Id : 89951411282 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A particular integral of $(D^2 + 3DD' + 2D'^2)z = x + y$ is

(a) $\frac{(x+y)^3}{6}$

(b) $\frac{(x+y)^3}{36}$

(c) $-\frac{(x+y)^3}{36}$

(d) $-\frac{(x+y)^3}{6}$

Options :

89951444298. 1

89951444299. 2

89951444300. 3

89951444301. 4

Question Number : 6 Question Id : 89951411283 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The PDE $u_{xx} - (1+y)^2 u_{yy} + \frac{1}{2}u_y + 4yu_x = 0$ is

(a) parabolic $\forall x \in \mathbb{R}, y \in \mathbb{R}$

(b) elliptic $\forall x \in \mathbb{R}, y \in \mathbb{R}$

(c) hyperbolic $\forall x \in \mathbb{R}, y \neq -1$

(d) hyperbolic $\forall x \in \mathbb{R}, y \geq 0$

Options :

89951444302. 1

89951444303. 2

89951444304. 3

89951444305. 4

Question Number : 7 Question Id : 89951411284 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The characteristic directions of the PDE $xyu_{xx} - (x^2 - y^2)u_{xy} - xyu_{yy} = 0$ are given by

(a) $x^2 - y^2 =$constant, $x/y =$ constant

(b) $x^2 + y^2 =$constant, $x^2/y =$ constant

(c) $x^2 - y^2 =$constant, $y/x =$ constant

(d) $x^2 + y^2 =$constant, $y/x =$ constant

Options :

89951444306. 1

89951444307. 2

89951444308. 3

89951444309. 4

Question Number : 8 Question Id : 89951411285 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A necessary and sufficient condition for the Pfaffian differential equation $\mathbf{X}.d\mathbf{r} = 0$, where $\mathbf{X} = (P(x,y,z), Q(x,y,z), R(x,y,z))$, $d\mathbf{r} = (dx, dy, dz)$ to be integrable is that

(a) $\nabla.(\nabla \times \mathbf{X}) = 0$

(b) $\nabla.\mathbf{X} = 0$

(c) $\nabla \times \mathbf{X} = 0$

(d) $\mathbf{X}.(\nabla \times \mathbf{X}) = 0$

Options :

89951444310. 1

89951444311. 2

89951444312. 3

89951444313. 4

Question Number : 9 Question Id : 89951411286 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Let ψ be harmonic in a domain V bounded by the closed surface S. If $\frac{\partial\psi}{\partial n} = 0$ on S, then

(a) $\frac{\partial\psi}{\partial n} = 0$ in V.

(b) $\psi = 0$ in V.

(c) ψ is unique or differ from one another by a constant in V.

(d) ψ is not unique in V.

Options :

89951444314. 1

89951444315. 2

89951444316. 3

89951444317. 4

Question Number : 10 Question Id : 89951411287 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Let $\psi(x,t)$ be a continuous function of x, t and is a solution of one dimensional heat equation $\psi_t = \psi_{xx}$ for $0 \leq x \leq L$, $0 \leq t \leq \tau$, where $\tau > 0$ is a fixed time. The function $\psi(x,t)$ attains its maximum and minimum values at

(a) $t = 0$ or $t = \tau$ or $x = 0$

(b) $t = 0$ or $t = \tau$ or $x = 0$ or $x = L$

(c) $t = 0$ or $x = 0$ or $x = L$

(d) $x = 0$ or $x = L$

Options :

89951444318. 1

89951444319. 2

89951444320. 3

89951444321. 4

Question Number : 11 Question Id : 89951411288 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The d'Alembert's solution of wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty \leq x \leq +\infty \quad ,t \geq 0$$

together with initial conditions $u(x,0) = \sin x$ and $\frac{\partial u}{\partial t}(x,0) = 0$ is given by

(a) $\sin x \cos t$

(b) $\cos x \sin t$

(c) $\sin x \sin t$

(d) $\cos x \cos t$

Options :
89951444322. 1
89951444323. 2
89951444324. 3
89951444325. 4

Question Number : 12 Question Id : 89951411289 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Solution of one dimensional heat equation $u_t = a^2 u_{xx}$ is given by

(a) $u(x,t) = e^{-a^2k^2t} + \sin kx$

(b) $u(x,t) = e^{a^2k^2t} \sin kx$

(c) $u(x,t) = e^{-a^2k^2t} \sin kx$

(d) $u(x,t) = e^{a^2k^2t} + \sin kx$

Options :
89951444326. 1
89951444327. 2
89951444328. 3
89951444329. 4

Question Number : 13 Question Id : 89951411290 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Laplace transform of e^{ax} is

(a) $\frac{s}{s-a}$

(b) $\frac{1}{s}$

(c) $\frac{1}{s+a}$

(d) $\frac{1}{s-a}$

Options :
89951444330. 1
89951444331. 2
89951444332. 3
89951444333. 4

Question Number : 14 Question Id : 89951411291 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

If the Fourier transform of $f(x)$ is $F(\omega)$, then the Fourier transform of $f^{(n)}(x)$ is given by

(a) $(+i\omega)^n F(\omega)$

(b) $(-i\omega)^n F(\omega)$

(c) $\omega^n F(\omega)$

(d) $(-\omega)^n F(\omega)$

where the superscript n denotes the nth order derivative of $f(x)$.

Options :

89951444334. 1

89951444335. 2

89951444336. 3

89951444337. 4

Question Number : 15 Question Id : 89951411292 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The general integral of the PDE $zpx - zqy = y^2 - x^2$ is given by

(a) $x^2 + y^2 + z^2 = f(xy)$

(b) $x^2 + y^2 = f(xyz)$

(c) $x + y + z = f(xyz)$

(d) none of the above

Options :

89951444338. 1

89951444339. 2

89951444340. 3

89951444341. 4

Question Number : 16 Question Id : 89951411293 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

If $x^2 + y^2 + z^2 = 1$, the value of $\frac{\partial z}{\partial y}$ at $(2/3, 1/3, 2/3)$ is equal to

(a) 1

(b) 0

(c) 1/2

(d) -1/2

Options :

89951444342. 1

89951444343. 2

89951444344. 3

89951444345. 4

Question Number : 17 Question Id : 89951411294 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Two first-order partial differential equations $F(x, y, z, p, q) = 0$ and $G(x, y, z, p, q) = 0$ are said to be compatible if

(a)

$$p\frac{\partial(F,G)}{\partial(x,p)} + q\frac{\partial(F,G)}{\partial(y,q)} + \frac{\partial(F,G)}{\partial(z,p)} + \frac{\partial(F,G)}{\partial(z,q)} = 0$$

(b)

$$p\frac{\partial(F,G)}{\partial(x,p)} + q\frac{\partial(F,G)}{\partial(y,q)} + \frac{\partial(F,G)}{\partial(x,z)} + \frac{\partial(F,G)}{\partial(y,z)} = 0$$

(c)

$$\frac{\partial(F,G)}{\partial(x,p)} + \frac{\partial(F,G)}{\partial(y,q)} + p\frac{\partial(F,G)}{\partial(x,z)} + q\frac{\partial(F,G)}{\partial(y,z)} = 0$$

(d)

$$\frac{\partial(F,G)}{\partial(x,p)} + \frac{\partial(F,G)}{\partial(y,q)} + p\frac{\partial(F,G)}{\partial(z,p)} + q\frac{\partial(F,G)}{\partial(z,q)} = 0$$

Options :

89951444346. 1

89951444347. 2

89951444348. 3
89951444349. 4

Question Number : 18 Question Id : 89951411295 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The Green's function $G(\mathbf{X}, \xi)$ of the non-homogeneous PDE $Lu(\mathbf{X}) = f(\mathbf{X})$, where $\mathbf{X} = (x, y, z)$ satisfies

(a) $LG(\mathbf{X}, \xi) = 0$ for $\mathbf{X} = \xi$

(b) $LG(\mathbf{X}, \xi) = 0$ for $\mathbf{X} \neq \xi$

(c) $LG(\mathbf{X}, \xi) = f(\mathbf{X})$ for $\mathbf{X} \neq \xi$

(d) $LG(\mathbf{X}, \xi) = f(\xi)$ for $\mathbf{X} \neq \xi$

Options :
89951444350. 1
89951444351. 2
89951444352. 3
89951444353. 4

Question Number : 19 Question Id : 89951411296 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The eigenvalues(λ_n) of the problem $\frac{d^2f}{dx^2} + \lambda^2 f = 0$ with $f(0) = f(a) = 0$ are given by

(a) $\lambda_n = \frac{n\pi}{a}$

(b) $\lambda_n = \frac{(2n+1)\pi}{a}$

(c) $\lambda_n = \frac{2n\pi}{a}$

(d) $\lambda_n = \frac{n^2\pi^2}{a^2}$

where n is an integer

Options :
89951444354. 1
89951444355. 2
89951444356. 3

89951444357. 4

Question Number : 20 Question Id : 89951411297 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

If $u(x,t) = e^{-i\omega x}\nu(t)$, with $\nu(0) = 1$ be a solution of $u_t = u_{xx}$, then

(a) $u = e^{-iw(x+i\omega t^2)}$

(b) $u = e^{-iw(x-i\omega t)}$

(c) $u = e^{-iw(x+i\omega t)}$

(d) $u = e^{-iw(x-i\omega t^2)}$

Options :

89951444358. 1

89951444359. 2

89951444360. 3

89951444361. 4

Partial Differential Equations-2

Section Id :	899514170
Section Number :	2
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	7
Number of Questions to be attempted :	5
Section Marks :	30
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	899514211
Question Shuffling Allowed :	Yes

Question Number : 21 Question Id : 89951411298 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Find the equation of the system of curves on the cylinder $2y = x^2$ orthogonal to its intersections with the hyperboloid of one-parameter system $xy = z + c$.

Question Number : 22 Question Id : 89951411299 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Find the general integral of the equation $(x - y)p + (y - x - z)q = z$ and the particular integral through the circle $z = 1, x^2 + y^2 = 1$.

Question Number : 23 Question Id : 89951411300 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Use Charpit's method to solve $(p^2 + q^2)y = qz$.

Question Number : 24 Question Id : 89951411301 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y) + e^{2x+y}$.

Question Number : 25 Question Id : 89951411302 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Solve $(x^2D^2 - y^2D'^2)z = \frac{x}{y}$.

Question Number : 26 Question Id : 89951411303 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Transform the Laplace equation in cartesian coordinates given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

in cylindrical co-ordinates (r, θ, z).

Question Number : 27 Question Id : 89951411304 Question Type : SUBJECTIVE
Correct Marks : 6

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Use Cole-Hopf transformation to the nonlinear Burgers' equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \quad x \in \mathbb{R}, t > 0$$

to make it linear.

Partial Differential Equations-3

Section Id :	899514171
Section Number :	3
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	7
Number of Questions to be attempted :	5
Section Marks :	50
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	899514212
Question Shuffling Allowed :	Yes

Question Number : 28 Question Id : 89951411305 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Classify and reduce the canonical form of the differential equation

$$e^x u_{xx} + e^y u_{yy} = u.$$

Question Number : 29 Question Id : 89951411306 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Use Lagrange's method to solve the first order PDE

$$p\cos(x+y) + q\sin(x+y) = z.$$

Question Number : 30 Question Id : 89951411307 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Use Laplace transform to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

subject to, $(i) u(0,t) = u(1,t) = 0, \quad t \geq 0$
$(ii) u(x,0) = \sin(\pi x), \frac{\partial u}{\partial t}(x,0) = -\sin(\pi x), \quad 0 \leq x \leq 1.$

Question Number : 31 Question Id : 89951411308 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Use separation of variables method to solve the one dimensional heat equation given by

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}, \quad 0 < x < l, \quad t > 0$$

subject to, $(i) \theta(0,t) = \theta(l,t) = 0, \quad t \geq 0$
$(ii) \theta(x,0) = x, \quad 0 \leq x \leq l.$

Question Number : 32 Question Id : 89951411309 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Define a harmonic function. Prove that if a harmonic function vanishes everywhere on the boundary of a domain, then it is identically zero everywhere. Also prove that if harmonic function defined in a bounded region satisfying Dirichlet condition exists, then it is unique.

Question Number : 33 Question Id : 89951411310 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Solve wave equation in spherical polar coordinates given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial u}{\partial r}), \quad r \geq 0, \quad t \geq 0.$$

Question Number : 34 Question Id : 89951411311 Question Type : SUBJECTIVE
Correct Marks : 10

The symbols p and q are used for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively.
The symbols D and D' are used for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively.

Find Green's function G for the Laplace equation $\nabla^2 u = 0$ in the half plane $x \geq 0$, $-\infty < y < \infty$, $-\infty < z < \infty$. Show that, G satisfies all necessary conditions. Hence find the solution of Laplace equation together with auxiliary conditions (i) $u = f(y, z)$ on $x = 0$ (ii) $u \to 0$ as $x \to \infty$.