

National Testing Agency

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Fourier Analysis and Its Applications

Group Number :	1
Group Id :	603489108
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Edit Attended Group? :	No
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Group Marks :	100
Is this Group for Examiner? :	No

Fourier Analysis and Its Applications-1

Section Id :	603489166
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory

Number of Questions :	75
Number of Questions to be attempted :	75
Section Marks :	100
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	603489294
Question Shuffling Allowed :	Yes

Question Number : 1 Question Id : 6034898115 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

Consider the 2π – periodic function

$$f(x) = \sum_{-\infty}^{\infty} \exp(-(x + 2\pi n)^2)$$

The constant term in the Fourier expansion of $f(x)$ equals:

- 1. 1
- 2. $\frac{1}{2\sqrt{\pi}}$
- 3. $\frac{1}{\sqrt{2\pi}}$
- 4. 2π

Options :

- 60348930435. 1
- 60348930436. 2
- 60348930437. 3
- 60348930438. 4

Question Number : 2 Question Id : 6034898116 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No
Correct Marks : 1 Wrong Marks : 0

The value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ equals:

1. $\pi^2/6$
2. $\pi^2/8$
3. $\pi^2/16$
4. $\pi^2/18$

Options :

60348930439. 1

60348930440. 2

60348930441. 3

60348930442. 4

Question Number : 3 Question Id : 6034898117 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Coefficients a_n, b_n in the Fourier series expansion of a 2π periodic function behave like n^{-1-c} , where c is a positive number. Then which of the following is true?

1. The series converges uniformly and absolutely
2. The series converges uniformly and but not absolutely
3. The series converges absolutely but not uniformly
4. The series converges neither uniformly nor absolute

Options :

60348930443. 1

60348930444. 2

60348930445. 3

60348930446. 4

Question Number : 4 Question Id : 6034898118 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Supply the value of c in Rodrigues formula for the Legendre polynomial $P_n(x) = c \frac{d^n}{dx^n} (x^2 - 1)^n$ equals:

1. 1
2. 2^n
3. $\frac{1}{2^n n!}$
4. $2^n n!$

Options :

60348930447. 1

60348930448. 2

60348930449. 3

60348930450. 4

Question Number : 5 Question Id : 6034898119 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

If $f(x)$ is in the Schwartz class and is an even function, then the Fourier transform of $\hat{f}(x)$ equals:

1. $f(x)$
2. $-f(x)$
3. $\pi f(x)$
4. $2\pi f(x)$

Options :

60348930451. 1

60348930452. 2

60348930453. 3

60348930454. 4

Question Number : 6 Question Id : 6034898120 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Given that $f(x) = \frac{1}{1+x^2}$ what is the value of $\hat{f}(0)$?

1. $\pi/2$
2. π
3. 2π
4. 1

Options :

60348930455. 1

60348930456. 2

60348930457. 3

60348930458. 4

Question Number : 7 Question Id : 6034898121 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is the expression for the Dirichlet kernel $D_n(x)$?

1. $\frac{1}{2\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin \frac{x}{2}}$
2. $\frac{1}{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin \frac{x}{2}}$
3. $\frac{\sin\left(n + \frac{1}{2}\right)x}{\sin \frac{x}{2}}$
4. $\frac{1}{2} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin \frac{x}{2}}$

Options :

60348930459. 1

60348930460. 2

60348930461. 3

60348930462. 4

Question Number : 8 Question Id : 6034898122 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following statement about the Volterra operator

$T : L^2[0, 1] \rightarrow L^2[0, 1]$ is false?

1. It is a compact operator
2. It is not a self-adjoint operator
3. It has no eigen-values
4. 0 is in the spectrum of this operator

Options :

60348930463. 1

60348930464. 2

60348930465. 3

60348930466. 4

Question Number : 9 Question Id : 6034898123 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Fejer's theorem asserts that for a continuous 2π periodic function $f(x)$, the nth partial sum $S_n(f, x)$

1. Converges uniformly to $f(x)$
2. Converges pointwise but not necessarily uniformly to $f(x)$
3. The Cesaro means of $S_n(f, x)$ converges uniformly to $f(x)$
4. The Cesaro means of $S_n(f, x)$ converges pointwise but not uniformly to $f(x)$

Options :

60348930467. 1

60348930468. 2

60348930469. 3

60348930470. 4

Question Number : 10 Question Id : 6034898124 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is true?

1. The Fejer and Poisson kernels are positive kernels
2. The Dirichlet and Poisson kernels are positive kernels
3. The Dirichlet and Fejer kernels are positive kernels
4. The Dirichlet, Poisson and Fejer kernels are positive kernels

Options :

60348930471. 1

60348930472. 2

60348930473. 3

60348930474. 4

Question Number : 11 Question Id : 6034898125 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Use the function $f(x)$ which equals 1 on the interval $[-1,1]$ and zero outside this interval to

compute the integral $\int_{-\infty}^{\infty} \frac{\sin^2 \xi}{\xi^2} d\xi$

1. $\frac{1}{\pi}$
2. $\frac{1}{2\pi}$
3. 2π
4. π

Options :

60348930475. 1

60348930476. 2

60348930477. 3

60348930478. 4

Question Number : 12 Question Id : 6034898126 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Parseval formula applied to the function $f(x) = |x|^2$ on $[-\pi, \pi]$ gives the value of $1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ as being equal to

1. $\frac{\pi^4}{64}$
2. $\frac{\pi^4}{80}$
3. $\frac{\pi^4}{90}$
4. $\frac{\pi^4}{96}$

Options :

60348930479. 1

60348930480. 2

60348930481. 3

60348930482. 4

Question Number : 13 Question Id : 6034898127 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The polynomials $\exp(x^2/2) \left(\frac{d}{dx} \right)^n \exp(-x^2/2)$ satisfy which one of the following equations?

1. $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$
2. $(1 - x^2)y'' - xy' + n^2y = 0$
3. $y'' - 2xy' + 2ny = 0$
4. $(1 - x^2)y'' + 2xy' + n(n + 1)y = 0$

Options :

60348930483. 1

60348930484. 2

60348930485. 3

60348930486. 4

Question Number : 14 Question Id : 6034898128 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Suppose $P_n(x)$ is the $n - th$ Legendre polynomial, then the value of $P_n'(1)$ equals

1. $n(n + 1)/2$
2. $n(n - 1)/2$
3. $n(n + 1)$
4. $n(n - 1)$

Options :

60348930487. 1

60348930488. 2

60348930489. 3

60348930490. 4

Question Number : 15 Question Id : 6034898129 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The Fourier transform of the function $f(x) = e^{-|x|}$ equals:

1. $2/(1 + \xi^2)a$
2. $1/(1 + \xi^2)a$
3. $1/2(1 + \xi^2)$
4. $1/4(1 + \xi^2)$

Options :

60348930491. 1

60348930492. 2

60348930493. 3

60348930494. 4

Question Number : 16 Question Id : 6034898130 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is not an eigenvalue of the Fourier transform as an operator on $L^2(\mathbb{R})$?

1. $\sqrt{\pi}$
2. $\sqrt{2\pi}$
3. $i\sqrt{2\pi}$
4. $-i\sqrt{2\pi}$

Options :

60348930495. 1

60348930496. 2

60348930497. 3

60348930498. 4

Question Number : 17 Question Id : 6034898131 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is the solution of the heat equation $u_t = u_{xx}$ with $u(x,0)=f(x)$? Here $f(x)$ is in the Schwartz class.

1. $\frac{1}{4\pi t} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t) dy$
2. $\frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t) dy$
3. $\frac{1}{\sqrt{\pi t}} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t) dy$
4. $\frac{1}{\sqrt{4t}} \int_{-\infty}^{\infty} f(y) \exp -((x-y)^2/4t) dy$

Options :

60348930499. 1

60348930500. 2

60348930501. 3

60348930502. 4

Question Number : 18 Question Id : 6034898132 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

What is the value of $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$?

Hint: Think of using the Laplace transform of $\int_{-\infty}^{\infty} \frac{\sin tx}{x} dx$ with respect to t .

1. $\frac{\pi}{2}$
2. 2π
3. $\sqrt{\pi}$
4. π

Options :

60348930503. 1

60348930504. 2

60348930505. 3

60348930506. 4

Question Number : 19 Question Id : 6034898133 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Consider the function $f(x) = 1 + \cos 3x + \sin^5 x$. The coefficients a, b of $\cos 6x$ and $\sin 5x$ in the Fourier series for $f(x)$ are, respectively,

1. $a = 1, b = 1/16$
2. $a = 1, b = 1/8$
3. $a = 0, b = 1/16$
4. $a = 0, b = 1/8$

Options :

60348930507. 1

60348930508. 2

60348930509. 3

60348930510. 4

Question Number : 20 Question Id : 6034898134 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Consider the function function $f(x) = x/|x|$, $f(0) = 0$ on $[-\pi, \pi]$.

Supply the value of the constant c in the following Fourier expansion

$$f(x) = \frac{c}{\pi} (\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots)$$

1. 2
2. 4
3. 1/2
4. 1/4

Options :

60348930511. 1

60348930512. 2

60348930513. 3

60348930514. 4

Question Number : 21 Question Id : 6034898135 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

For which of the following kernels $k(x, y)$ is the associated integral operator on $L^2[0, 1]$ compact and self adjoint?

1. $k(x, y) = 1$, if $x < y$ and $k(x, y) = 0$, if $x > y$
2. $k(x, y) = 0$, if $x < y$ and $k(x, y) = 1$, if $x > y$
3. $k(x, y) = \frac{\sin(2x - y)}{2x - y}$
4. $k(x, y) = \frac{\sin(x - y)}{x - y}$

Options :

60348930515. 1

60348930516. 2

60348930517. 3

60348930518. 4

Question Number : 22 Question Id : 6034898136 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The Lebesgue constants are the numbers $L_n = \int_0^\pi |D_n(t)| dt$ where $D_n(t)$ is the Dirichlet kernel.

Which of the following is TRUE?

1. $\sum_{n=2}^{\infty} L_n$ converges
2. $\sum_{n=2}^{\infty} L_n^{-1}$ converges
3. $\sum_{n=2}^{\infty} \frac{1}{nL_n^2}$ converges
4. $\sum_{n=2}^{\infty} \frac{1}{nL_n}$ converges

Options :

60348930519. 1
60348930520. 2
60348930521. 3
60348930522. 4

Question Number : 23 Question Id : 6034898137 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Consider the solution $u(x, y)$ of $\Delta u = 0$ on $\{(x, y): x^2 + y^2 < 1\}$ and

$u(\cos \theta, \sin \theta) = |\cos \theta|$. Then the value of $u(0, 0)$ equals:

1. $\frac{\pi}{2}$
2. $1/\pi$
3. $2/\pi$
4. π

Options :

60348930523. 1

60348930524. 2

60348930525. 3

60348930526. 4

Question Number : 24 Question Id : 6034898138 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The value of $\int_0^{2\pi} \frac{\left(1 - \frac{1}{9}\right) d\theta}{1 + \frac{1}{9} - \frac{2}{9} \cos \theta}$ equals:

1. $\frac{\pi}{4}$
2. $\frac{\pi}{2}$
3. π
4. 2π

Options :

60348930527. 1

60348930528. 2

60348930529. 3

60348930530. 4

Question Number : 25 Question Id : 6034898139 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

S is a closed bounded subset of $C[0, 1]$ endowed with the supremum norm. To apply the Ascoli-Arzelà theorem, which additional conditions are needed on S ?

1. S consists of non-negative functions
2. S is equi-continuous
3. Every member of S is Lipschitz continuous
4. No additional hypothesis is needed

Options :

60348930531. 1

60348930532. 2

60348930533. 3

60348930534. 4

Question Number : 26 Question Id : 6034898140 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Given that $x^4 = c_4P_4(x) + c_3P_3(x) + \dots + c_0P_0(x)$ where $P_n(x)$ is the n - th Legendre polynomial, the value of c_4 equals:

1. 8/15

2. 8/25

3. 8/35

4. 8/45

Options :

60348930535. 1

60348930536. 2

60348930537. 3

60348930538. 4

Question Number : 27 Question Id : 6034898141 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

What is the value of $2 \int_0^1 P_4(x)dx$? Here $P_n(x)$ is the n -th Legendre polynomial.

1. 0

2. 3/16

3. -3/16

4. 3/8

Options :

60348930539. 1

60348930540. 2

60348930541. 3

60348930542. 4

Question Number : 28 Question Id : 6034898142 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Supply the constant α in the following formula:

$$\alpha \int_{-\pi}^{\pi} |f(x)|^2 dx = |a_0|^2 + \frac{1}{2} \sum_1^{\infty} (|a_n|^2 + |b_n|^2)$$

1. $1/\pi$
2. $2/\pi$
3. 2
4. $1/(2\pi)$

Options :

60348930543. 1

60348930544. 2

60348930545. 3

60348930546. 4

Question Number : 29 Question Id : 6034898143 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The constant term in the Fourier series expansion of $f(x) = x^4$ on the interval $[-\pi, \pi]$ equals:

1. $\pi^3/5$
2. $\pi^3/10$
3. $\pi^4/5$
4. $\pi^4/10$

Options :

60348930547. 1

60348930548. 2

60348930549. 3

60348930550. 4

Question Number : 30 Question Id : 6034898144 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

What is the value of the limit $\lim_n \frac{\sqrt[n]{(2n)!}}{n^2}$?

1. $1/e^2$
2. $4/e^2$
3. $1/e$
4. $4/e$

Options :

60348930551. 1

60348930552. 2

60348930553. 3

60348930554. 4

Question Number : 31 Question Id : 6034898145 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The $n - th$ Legendre polynomial is $P_n(x)$. Then the value of $P_5'(0)$ equals

1. $15/4$
2. $15/8$
3. $15/16$
4. $15/32$

Options :

60348930555. 1

60348930556. 2

60348930557. 3

60348930558. 4

Question Number : 32 Question Id : 6034898146 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Given that $f(x) = (1+x^2)^{-2}$, what is the value of $\hat{f}(0)$?

1. $\frac{\pi}{2}$
2. π
3. 2π
4. 4π

Options :

60348930559. 1
60348930560. 2
60348930561. 3
60348930562. 4

Question Number : 33 Question Id : 6034898147 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is FALSE about the Fourier transform as an operator from $L^2(\mathbb{R})$ to itself?

1. It is invertible
2. It is bounded
3. It is unitary
4. It is self-adjoint

Options :

60348930563. 1
60348930564. 2
60348930565. 3
60348930566. 4

Question Number : 34 Question Id : 6034898148 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The Lebesgue constants are the numbers $L_n = \int_0^\pi |D_n(t)| dt$, where $D_n(t)$ is the Dirichlet kernel. Which of the following is FALSE?

1. $\sum_{n=2}^{\infty} \frac{1}{n(\log L_n)}$ diverges
2. $\sum_{n=2}^{\infty} \frac{1}{n^2(\log L_n)}$ converges
3. $\sum_{n=2}^{\infty} \frac{1}{n^2(\log L_n)}$ diverges
4. $\sum_{n=2}^{\infty} \frac{1}{n(\log L_n)^2}$ converges

Options :

60348930567. 1

60348930568. 2

60348930569. 3

60348930570. 4

Question Number : 35 Question Id : 6034898149 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Fourier transform of $\exp(-x^2)$ is $a \exp(-\frac{x^2}{4})$. Then the value of a equals

1. $\sqrt{\pi}$
2. $\sqrt{2\pi}$
3. $2\sqrt{\pi}$
4. $4\sqrt{\pi}$

Options :

60348930571. 1

60348930572. 2

60348930573. 3

60348930574. 4

Question Number : 36 Question Id : 6034898150 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which of the following is a trigonometric polynomial on $[-\pi, \pi]$?

1. $\sin \frac{x}{5}$
2. $\sin^5 x$
3. $\sin^{\frac{1}{5}} x$
4. $\sin \sqrt{5} x$

Options :

60348930575. 1
60348930576. 2
60348930577. 3
60348930578. 4

Question Number : 37 Question Id : 6034898151 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Which of the following is FALSE?

1. Trigonometric polynomials are dense in $L^2[-\pi, \pi]$ with respect to L^2 norm
2. Trigonometric polynomials are dense in $L^1[-\pi, \pi]$ with respect to L^1 norm
3. Trigonometric polynomials are dense in the closed subspace $S = \{f \in C[-\pi, \pi] : f(-\pi) = f(\pi)\}$ of $C[-\pi, \pi]$ with supremum norm
4. Trigonometric polynomials are dense in $L^\infty[-\pi, \pi]$ with L^∞ norm

Options :

60348930579. 1
60348930580. 2
60348930581. 3
60348930582. 4

Question Number : 38 Question Id : 6034898152 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Let $f(x) = (1 - x^2)^{-1/2}$ on $(-1, 1)$ and zero outside. The Fourier transform of $f(x)$ is

1. $2\pi J_0(\xi)$
2. $\pi J_0(\xi)$
3. $\pi J_0(\xi)/2$
4. $J_0(\xi)$

Options :

60348930583. 1

60348930584. 2

60348930585. 3

60348930586. 4

Question Number : 39 Question Id : 6034898153 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Given that $f(x) = \sin^2 x$ on $[-\pi, \pi]$ and zero outside, $g(x) = \cos^2 x$ on $[-\pi, \pi]$ and zero outside, $[-\pi, \pi]$. Then what is the value of $\int_{-\infty}^{\infty} f * g(x) dx$?

Note $f * g$ is the convolution of f and g

1. π^2
2. $2\pi^2$
3. 2π
4. π

Options :

60348930587. 1

60348930588. 2

60348930589. 3

60348930590. 4

Question Number : 40 Question Id : 6034898154 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Given that the Fourier transform of a Schwartz function $f(x)$ is $F(\xi)$. The Fourier transform of $F'(\xi)$ equals

1. $2\pi i f(-x)$
2. $2\pi i x f(x)$
3. $2\pi i x f(x)$
4. $2\pi i x f(-x)$

Options :

60348930591. 1
60348930592. 2
60348930593. 3
60348930594. 4

Question Number : 41 Question Id : 6034898155 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

A sequence of functions $f_n(x)$ in $L^2[0,1]$ converges in norm then which of the following is false?

1. It converges weakly
2. It is norm bounded
3. The sequence of arithmetic means converges weakly
4. Converges pointwise

Options :

60348930595. 1
60348930596. 2
60348930597. 3
60348930598. 4

Question Number : 42 Question Id : 6034898156 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is an odd function?

1. $J_1'(x)$
2. $P_5'(x)$
3. Fourier transform of an odd function
4. Fourier transform of an even function

Options :

60348930599. 1

60348930600. 2

60348930601. 3

60348930602. 4

Question Number : 43 Question Id : 6034898157 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is Luzin's theorem?

1. $C[0,1]$ is dense in $L^2[0,1]$ with respect to L^1 norm
2. Polynomials are dense in $C[0,1]$
3. Polynomials are dense in $L^1[0,1]$
4. $C[0,1]$ is dense in $L^\infty[0,1]$ with respect to L^∞ norm

Options :

60348930603. 1

60348930604. 2

60348930605. 3

60348930606. 4

Question Number : 44 Question Id : 6034898158 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Fourier transform of a radial function is

1. Always radial and given by a sine transform in two dimensions
2. Always radial and given by a sine transform in three dimensions
3. Always radial and given by a Bessel transform in three dimensions
4. Always radial and given by a cosine transform in two dimensions

Options :

60348930607. 1

60348930608. 2

60348930609. 3

60348930610. 4

Question Number : 45 Question Id : 6034898159 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

For the heat equation $u_t - u_{xx} = 0$ with initial condition $u(x,0) = f(x)$. Assume that $f(x)$ is positive on $(-1,1)$ and zero outside. Then

1. $u(x,t)$ is positive for all $x \in \mathbb{R}$ and for all positive t
2. $u(x, t)$ is positive only for all $x \in (-1, 1)$ and for all positive t
3. $u(x,t)$ is positive only for all $x \in (-2,2)$ and for all positive t
4. $u(x,t)$ is positive only for all $x \in (0,\infty)$ and for all positive t

Options :

60348930611. 1

60348930612. 2

60348930613. 3

60348930614. 4

Question Number : 46 Question Id : 6034898160 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

For the wave equation $u_{tt} - u_{xx} = 0$ with initial conditions in the Schwartz class, assume that $E(t)$ is the total energy and $\Pi(t)$ is the kinetic energy. Then, which of the following is true?

1. $\Pi(t)$ is constant in time
2. $\Pi(t)$ necessarily decays to zero as time approaches infinity
3. $\Pi(t)$ approaches $E(0)/2$ as time goes to infinity
4. $\Pi(t)$ approaches $E(0)$ as time goes to infinity

Options :

60348930615. 1

60348930616. 2

60348930617. 3

60348930618. 4

Question Number : 47 Question Id : 6034898161 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The sum $1 + 2\cos x + 2\cos 2x + \dots + 2\cos nx$ equals:

1. $\frac{\sin(nx+x)}{\sin \frac{x}{2}}$
2. $\frac{\sin(nx+\frac{x}{2})}{\sin \frac{x}{2}}$
3. $\frac{\sin(nx+\frac{x}{2})}{\sin x}$
4. $\frac{\sin x}{\sin(nx)}$

Options :

60348930619. 1

60348930620. 2

60348930621. 3

60348930622. 4

Question Number : 48 Question Id : 6034898162 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Which of the following is false about the n -th Legendre polynomial?

1. It has a double root at a point in $(-1,1)$
2. It has n distinct roots in $(-1,1)$
3. It does not vanish at 1
4. For odd n it vanishes at the origin

Options :

60348930623. 1

60348930624. 2

60348930625. 3

60348930626. 4

Question Number : 49 Question Id : 6034898163 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Suppose T_n is a sequence of continuous linear maps from a Banach space X into $\mathbb{R}$ such that for each $x \in X$ the sequence $T_n(x)$ is bounded, then which of the following is false?

1. $\|T_n\|$ is bounded
2. $\|T_n(x)\|$ has a bound independent of x for all $\|x\| \leq 1$
3. $\|T_n\|$ converges
4. $\|T_n\|$ has a convergent subsequence.

Options :

60348930627. 1

60348930628. 2

60348930629. 3

60348930630. 4

Question Number : 50 Question Id : 6034898164 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The Banach Steinhaus's theorem implies

1. Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series converges uniformly
2. Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series converges pointwise
3. Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series diverges at a specific point.
4. Existence of a continuous function on $[-\pi, \pi]$ whose Fourier series diverges everywhere.

Options :

60348930631. 1

60348930632. 2

60348930633. 3

60348930634. 4

Sub-Section Number :

2

Sub-Section Id :

603489295

Question Shuffling Allowed :

Yes

Question Number : 51 Question Id : 6034898165 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which of the following is not a trigonometric polynomial on $[-\pi, \pi]$?

1. $\cos 3x$
2. $\cos^{1/3} x$
3. $\cos^3 x$
4. $3 \cos x$

Options :

60348930635. 1

60348930636. 2

60348930637. 3

60348930638. 4

Question Number : 52 Question Id : 6034898166 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Supply the value of the constant c in the formula $\hat{f}(x) = 2\pi f(cx)$, where $f(x)$ is in the Schwartz's class.

1. -1
2. 1
3. -2
4. 2

Options :

60348930639. 1

60348930640. 2

60348930641. 3

60348930642. 4

Question Number : 53 Question Id : 6034898167 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which of the following about the Bessel's function $J_p(x)$ is true?

1. It is an entire function of z for any value of p
2. It is an entire function of z for no value of p
3. It is an entire function of z when $p = k + \frac{1}{2}$ where k is a natural number
4. It is an entire function of z for any non-negative integer value of p

Options :

60348930643. 1

60348930644. 2

60348930645. 3

60348930646. 4

Question Number : 54 Question Id : 6034898168 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The Weierstrass's approximation theorem implies

1. The separability of $C[0,1]$
2. The separability of $L^\infty[0,1]$
3. That $C[0,1]$ is not separable
4. That $L^1[0,1]$ is not separable

Options :

60348930647. 1

60348930648. 2

60348930649. 3

60348930650. 4

Question Number : 55 Question Id : 6034898169 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which of the following is the Bessel's equation of order p ?

1. $x^2 y'' - xy' + (x^2 + p^2)y = 0$
2. $x^2 y'' - xy' + (x^2 - p^2)y = 0$
3. $x^2 y'' + xy' + (x^2 + p^2)y = 0$
4. $x^2 y'' + xy' + (x^2 - p^2)y = 0$

Options :

60348930651. 1

60348930652. 2

60348930653. 3

60348930654. 4

Question Number : 56 Question Id : 6034898170 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

For vectors v and w in a Hilbert space, given that $\|v\| = 4$, $\|w\| = 5$ and $\|w+v\| = 3$. Then, what is the value of $\|w-v\|$?

1. $\sqrt{63}$
2. $\sqrt{69}$
3. $\sqrt{71}$
4. $\sqrt{73}$

Options :

60348930655. 1

60348930656. 2

60348930657. 3

60348930658. 4

Question Number : 57 Question Id : 6034898171 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

If $F(\xi)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f'(x)$ equals

1. $\xi F(\xi)$
2. $-\xi F(\xi)$
3. $i\xi F(\xi)$
4. $-i\xi F(\xi)$

Options :

60348930659. 1

60348930660. 2

60348930661. 3

60348930662. 4

Question Number : 58 Question Id : 6034898172 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

If $F(\xi)$ is the Fourier transform of $f(x)$ then, the Fourier transform of $ixf(x)$ equals:

1. $-iF'(\xi)$
2. $iF'(\xi)$
3. $-F'(\xi)$
4. $F'(\xi)$

Options :

60348930663. 1

60348930664. 2

60348930665. 3

60348930666. 4

Question Number : 59 Question Id : 6034898173 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which one of the following is false?

1. $L^1[-\pi, \pi]$ with respect to L^1 norm has uncountable Hamel basis
2. $L^1[-\pi, \pi]$ with respect to L^1 norm has countable Hamel basis
3. The set of all polynomials has countable Hamel basis
4. $L^\infty[-\pi, \pi]$ with respect to L^∞ norm has uncountable Hamel basis

Options :

60348930667. 1

60348930668. 2

60348930669. 3

60348930670. 4

Question Number : 60 Question Id : 6034898174 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Let $y(x)$ be a solution of the differential equation $y'' + x^2y = 0$.

Then, which one of the following is true?

1. The zeros of $y(x)$ form a finite set
2. The zeros of $y(x)$ are equally spaced
3. The zeros (z_n) of $y(x)$ are countable discrete increasing sequence and $|z_n - z_{n-1}|$ tends to zero as n tends to infinity
4. The zeros of $y(x)$ have a finite limit point

Options :

60348930671. 1

60348930672. 2

60348930673. 3

60348930674. 4

Question Number : 61 Question Id : 6034898175 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which one of the following is an essential hypothesis on a metric space in order to apply the Baire Category theorem?

1. X is complete
2. X is locally connected
3. X is connected
4. X is separable

Options :

60348930675. 1

60348930676. 2

60348930677. 3

60348930678. 4

Question Number : 62 Question Id : 6034898176 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Given that F is the Fourier transform of a Schwartz function f . Supply the value of c in the

following formula $c \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\hat{f}(\xi)|^2 d\xi$

1. $\frac{1}{2\pi}$
2. $\frac{1}{\sqrt{2\pi}}$
3. $\sqrt{2\pi}$
4. 2π

Options :

60348930679. 1

60348930680. 2

60348930681. 3

60348930682. 4

Question Number : 63 Question Id : 6034898177 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Given that $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ which of the following does not always hold?

Here $F(\xi)$ is the Fourier transform of f

1. $F(\xi)$ is uniformly continuous
2. $F(\xi) \in L^1(\mathbb{R})$
3. $F(\xi)$ tends to zero as ξ tends to infinity
4. $F(\xi) \in L^2(\mathbb{R})$

Options :

60348930683. 1

60348930684. 2

60348930685. 3

60348930686. 4

Question Number : 64 Question Id : 6034898178 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which of the following is the recurrence relation for the power series solution of Legendre's equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$

1. $\frac{a_{n+2}}{a_n} = \frac{n(n+1) + p(p+1)}{(n+1)(n+2)}$
2. $\frac{a_{n+2}}{a_n} = \frac{n(n+1) - p(p+1)}{(n+1)(n+2)}$
3. $\frac{a_{n+2}}{a_n} = \frac{n(n-1) - p(p-1)}{(n+1)(n+2)}$
4. $\frac{a_{n+2}}{a_n} = \frac{n(n-1) + p(p-1)}{(n+1)(n+2)}$

Options :

60348930687. 1

60348930688. 2

60348930689. 3

60348930690. 4

Question Number : 65 Question Id : 6034898179 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The function $|x|^{\frac{3}{2}}$ defined on $[-\pi, \pi]$ and extended as a 2π periodic function is

1. Holder continuous but not Lipschitz continuous
2. Continuously differentiable
3. Not Lipschitz continuous
4. Lipschitz continuous but not continuously differentiable

Options :

60348930691. 1

60348930692. 2

60348930693. 3

60348930694. 4

Question Number : 66 Question Id : 6034898180 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The Fourier series of the function $|x|^5$ on $[-\pi, \pi]$ converges

1. Absolutely but not uniformly
2. Absolutely and uniformly
3. Uniformly but not absolutely
4. Conditionally and non-uniformly

Options :

60348930695. 1

60348930696. 2

60348930697. 3

60348930698. 4

Question Number : 67 Question Id : 6034898181 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

To conclude that a compact operator has an orthonormal basis of eigenvectors, which of the following hypothesis is sufficient?

1. The operator is self-adjoint
2. The operator is normal
3. The operator is unitary
4. The operator is positive

Options :

60348930699. 1

60348930700. 2

60348930701. 3

60348930702. 4

Question Number : 68 Question Id : 6034898182 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Consider the function on $f(x) = \pi - |x|$ on $(-\pi, \pi)$. The Fourier series for the function is

1. $\pi + 4 \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{\pi(2n-1)^2}$
2. $\pi + 2 \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{\pi(2n-1)^2}$
3. $\frac{\pi}{2} + 4 \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{\pi(2n-1)^2}$
4. $\frac{\pi}{2} + 2 \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{\pi(2n-1)^2}$

Options :

60348930703. 1

60348930704. 2

60348930705. 3

60348930706. 4

Question Number : 69 Question Id : 6034898183 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

By computing the Fourier series for x^2 or otherwise, the value of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ equals:

1. $\frac{\pi^2}{12}$
2. $\frac{\pi^2}{18}$
3. $\frac{\pi^2}{24}$
4. $\frac{\pi^2}{30}$

Options :

60348930707. 1

60348930708. 2

60348930709. 3

60348930710. 4

Question Number : 70 Question Id : 6034898184 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The coefficient of $\cos 2nx$ in the Fourier series of $\cos ax$ (a is not an integer) is:

1. $-\sin a\pi/(\pi(4n^2 - a^2))$
2. $\sin a\pi/(\pi(4n^2 - a^2))$
3. $2a \sin a\pi/(\pi(4n^2 - a^2))$
4. $-2a \sin a\pi/(\pi(4n^2 - a^2))$

Options :

60348930711. 1

60348930712. 2

60348930713. 3

60348930714. 4

Question Number : 71 Question Id : 6034898185 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Which one of the following is a periodic function?

1. $\cos x + \sin \sqrt{5} x$
2. $\sin 5x + \cos \sqrt{7} x$
3. $\sin 7x + \sin \sqrt{7} x$
4. $\sin \sqrt{7} x + \sin 7\sqrt{7} x$

Options :

60348930715. 1

60348930716. 2

60348930717. 3

60348930718. 4

Question Number : 72 Question Id : 6034898186 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The Fourier transform of a certain L^1 function on the real line fails to be in L^1 (on the real line). Which of the following is a possible candidate for this function?

1. $\chi_{[-1,1]}$
2. $\chi_{[-1,1]} * \chi_{[-1,1]}$
3. $\chi_{[-1,1]} * \chi_{[-1,1]} * \chi_{[-1,1]}$
4. e^{-x^2}

Options :

60348930719. 1

60348930720. 2

60348930721. 3

60348930722. 4

Question Number : 73 Question Id : 6034898187 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

Let $f(x) = \frac{1}{\sqrt{\pi^2 - x^2}}$, then

1. $f \in L^1[-\pi, \pi]$ but, $f \notin L^2[-\pi, \pi]$
2. $f \in L^1[0,1] \cap L^2[0,1]$
3. $f \in C[-\pi, \pi]$
4. $f \notin L^1[-\pi, \pi]$

Options :

60348930723. 1

60348930724. 2

60348930725. 3

60348930726. 4

Question Number : 74 Question Id : 6034898188 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The coefficient of $\cos x$ in the Fourier expansion of $f(x) = \pi^2 - x^2$ equals

1. -2
2. 2
3. -4
4. 4

Options :

60348930727. 1

60348930728. 2

60348930729. 3

60348930730. 4

Question Number : 75 Question Id : 6034898189 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 2 Wrong Marks : 0

The Fourier transform of $\frac{1}{1+x^2}$ is $g(x)$. Which of the following is FALSE?

1. $g(x)$ is an even function
2. $g(x)$ is an odd function
3. $g(x)$ is in L^1 (on the real line)
4. $g(x)$ is in L^2 (on the real line)

Options :

60348930731. 1

60348930732. 2

60348930733. 3

60348930734. 4