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National Testing Agency

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Partial Differential Equations

Group Number :	1
Group Id :	864351130
Group Maximum Duration :	0
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Show Attended Group? :	No
Edit Attended Group? :	No
Break time :	0
Group Marks :	100
Is this Group for Examiner? :	No

Partial Differential Equations-1

Section Id :	864351548
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	20

Number of Questions to be attempted : 20
Section Marks : 20
Mark As Answered Required? : Yes
Sub-Section Number : 1
Sub-Section Id : 864351594
Question Shuffling Allowed : Yes

Question Number : 1 Question Id : 86435111093 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The partial differential equation $u_t + cuu_x + u_{xxx} = 0$ is

1. A first-order nonlinear equation
2. A third-order quasi-linear equation
3. A third-order linear equation
4. A first-order quasi linear equation

Options :
86435136323. 1
86435136324. 2
86435136325. 3
86435136326. 4

Question Number : 2 Question Id : 86435111094 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A mathematical problem is said to be well-posed if

1. Unique solution exists
2. At least one solution exists and the solution depends continuously on the data
3. At most one solution exists and the solution depends continuously on the data
4. Solution depends continuously on the data

Options :
86435136327. 1

86435136328. 2
86435136329. 3
86435136330. 4

Question Number : 3 Question Id : 86435111095 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The function $x^2 - y^2$ is a solution of the equation

1. $u_{xx} + u_{yy} = 0$
2. $u_{xx} - u_{yy} = 0$
3. $u_{xx} + u_{xy} = 0$
4. $u_{yy} + u_{xy} = 0$

Options :
86435136331. 1
86435136332. 2
86435136333. 3
86435136334. 4

Question Number : 4 Question Id : 86435111096 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The PDE formed by eliminating arbitrary functions from the equation

$u(x, y) = b(y) - a(y)e^{-x}$ is

1. $u_{xy} + u_y = 0$
2. $u_{xy} + u_x = 0$
3. $u_{yy} + u_y = 0$
4. $u_{xx} + u_x = 0$

Options :
86435136335. 1

86435136336. 2
86435136337. 3
86435136338. 4

Question Number : 5 Question Id : 86435111097 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The first order PDE formed by eliminating arbitrary constants a and b from the equation $2z(x,y) = (ax+y)^2 + b$ is given by

1. $px + qy = p^2$
2. $px + qy = q^2$
3. $px - qy = q^2$
4. $px - qy = p^2$

Options :
86435136339. 1
86435136340. 2
86435136341. 3
86435136342. 4

Question Number : 6 Question Id : 86435111098 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The general solution of $2p + 3q = 1$ is given by

1. $3z - y = f(3x - 2y)$
2. $3z - x = f(3x + 2y)$
3. $2x - 3y = f(z)$
4. $2x - 3y = f(z - 3y)$

Options :
86435136343. 1

86435136344. 2
86435136345. 3
86435136346. 4

Question Number : 7 Question Id : 86435111099 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The primitive of the Paffian differential equation
$y(y+z)d(x+z)+(y+z)(z+x)dy-y(z+x)d(y+z)=0$ is given by

(c is an arbitrary constant)

1. $x(y+z)=c(x+y)$
2. $y(z+x)=c(y+z)$
3. $z(x+y)=c(z+x)$
4. None of the above

Options :
86435136347. 1
86435136348. 2
86435136349. 3
86435136350. 4

Question Number : 8 Question Id : 86435111100 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The complete integral of $p+q=pq$ is given by

1. $z=\frac{a}{a-1}y+bx+a$
2. $z=ax+by$
3. $z=\frac{a}{a-1}x+ay+b$
4. None of the above

Options :

86435136351. 1
86435136352. 2
86435136353. 3
86435136354. 4

Question Number : 9 Question Id : 86435111101 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The general solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$ is given by

1. $z = \phi_1(y+x) + e^{-x}\phi_2(y-x)$
2. $z = \phi_1(y-x) + e^{-x}\phi_2(y+x)$
3. $z = \phi_1(y-x) + e^{x}\phi_2(y+x)$
4. $z = \phi_1(y+x) + e^{x}\phi_2(y-x)$

Options :
86435136355. 1
86435136356. 2
86435136357. 3
86435136358. 4

Question Number : 10 Question Id : 86435111102 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A particular integral of $(D^2 - DD' - 2D)z = e^{2x+y}$ is given by

1. $z = -\frac{e^{2x+y}}{4}$
2. $z = \frac{e^{2x+y}}{2}$
3. $z = -\frac{e^{2x+y}}{2}$
4. None of the above

Options :
86435136359. 1
86435136360. 2
86435136361. 3
86435136362. 4

Question Number : 11 Question Id : 86435111103 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The PDE $\sin^2 x\, Z_{xx} + \sin 2x\, Z_{xy} + \cos^2 x\, Z_{yy} = 0$ is

1. Parabolic $\forall x \in R \; and \; y = 0$
2. Elliptic $\forall x \in R \; and \; y \in R$
3. Hyperbolic $\forall x \in R \; and \; y \in R$
4. Parabolic $\forall x \in R \; and \; y \in R$

Options :
86435136363. 1
86435136364. 2
86435136365. 3
86435136366. 4

Question Number : 12 Question Id : 86435111104 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The two characteristic directions for the PDE
$xy\, u_{xx} - (x^2 - y^2)u_{xy} - xyu_{yy} + yu_x - xu_y = 2(x^2 - y^2)$ are given by

1. $\frac{dy}{dx} = x, \quad \frac{dy}{dx} = \frac{x^2 - y^2}{xy}$
2. $\frac{dy}{dx} = \frac{y}{x}, \quad \frac{dy}{dx} = -\frac{x}{y}$
3. $\frac{dy}{dx} = -\frac{y}{x}, \quad \frac{dy}{dx} = \frac{x}{y}$
4. $\frac{dy}{dx} = y, \; \frac{dy}{dx} = \frac{x^2 - y^2}{xy}$

Options :
86435136367. 1
86435136368. 2
86435136369. 3
86435136370. 4

Question Number : 13 Question Id : 86435111105 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The complementary function for $(xyDD' - y^2D'^2 - 3xD + 2yD')u = 0$ is given by

1. $u = y^3\phi_1(x) + \phi_2(xy)$
2. $u = x^3\phi_1(y) + \phi_2(xy)$
3. $u = y^3\phi_1(x) + \phi_2(\frac{y}{x})$
4. $u = x^3\phi_1(y) + \phi_2(\frac{y}{x})$

Options :
86435136371. 1
86435136372. 2
86435136373. 3
86435136374. 4

Question Number : 14 Question Id : 86435111106 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Consider the linear hyperbolic equation $L(z) = f(x, y)$, where $L(z) = \frac{\partial^2 z}{\partial x \partial y} + c(x, y)z$.

Suppose

$wL(z) - zL(w) = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}$, in which w is an arbitrary function of x and y having continuous derivatives. Then the expressions for U and V are

1. $U = z\frac{\partial w}{\partial y}, \quad V = -w\frac{\partial z}{\partial x}$
2. $U = -w\frac{\partial z}{\partial x}, \quad V = z\frac{\partial w}{\partial y}$
3. $U = w\frac{\partial z}{\partial x}, \quad V = -z\frac{\partial w}{\partial y}$
4. $U = -z\frac{\partial w}{\partial y}, \quad V = w\frac{\partial z}{\partial x}$

Options :
86435136375. 1
86435136376. 2
86435136377. 3
86435136378. 4

Question Number : 15 Question Id : 86435111107 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The solution of the Cauchy problem $u_{tt} - c^2 u_{xx} = 0, -\infty < x < \infty, t \geq 0,\ u(x,0) = 0, u_t(x,0) = 1$ is

1. $u(x,t) = x - ct$
2. $u(x,t) = t$
3. $u(x,t) = x$
4. $u(x,t) = x + ct$

Options :
86435136379. 1
86435136380. 2

86435136381. 3
86435136382. 4

Question Number : 16 Question Id : 86435111108 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

Laplace equation in spherical polar coordinates (r, θ, ϕ) is given by

1. $\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2} = 0$
2. $\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{sin\theta}\frac{\partial^2 u}{\partial\phi^2} = 0$
3. $\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin^2\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2} = 0$
4. $\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2} = 0$

Options :
86435136383. 1
86435136384. 2
86435136385. 3
86435136386. 4

Question Number : 17 Question Id : 86435111109 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The Fourier transform of e^{-ax^2}, $a > 0$ is

1. $\frac{1}{\sqrt{2a}}\, e^{-\frac{p^2}{4a^2}}$
2. $\frac{1}{\sqrt{2}\, a}\, e^{-\frac{p^2}{4a^2}}$
3. $\frac{1}{\sqrt{2a}}\, e^{-\frac{p^2}{4a}}$
4. None of the above

Options :
86435136387. 1
86435136388. 2
86435136389. 3
86435136390. 4

Question Number : 18 Question Id : 86435111110 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The Laplace transform of 1 is

1. $\frac{1}{p}$
2. $\frac{1}{p^2}$
3. p
4. p^2

Options :
86435136391. 1
86435136392. 2
86435136393. 3
86435136394. 4

Question Number : 19 Question Id : 86435111111 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

The dispersion relation for water waves in an ocean of depth h_0 is

1. $w^2 = gk\,\mathrm{sech}\,kh$
2. $w^2 = gk\tanh kh$
3. $w^2 = gkh$
4. None of the above

Options :
86435136395. 1

86435136396. 2
86435136397. 3
86435136398. 4

Question Number : 20 Question Id : 86435111112 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No Correct Marks : 1 Wrong Marks : 0

A bounded solution of the PDE $u_t = u_{xx} + e^{-t}$ is

1. $u = x - e^{-t}$
2. $u = e^{-x}e^{-t}$
3. $u = x - e^{-x}e^{t}$
4. $u = e^{x}e^{-t}$

Options :
86435136399. 1
86435136400. 2
86435136401. 3
86435136402. 4

Partial Differential Equations-2

Section Id :	864351549
Section Number :	2
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	10
Section Marks :	30
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	864351595
Question Shuffling Allowed :	No

Question Number : 21 Question Id : 86435111113 Question Type : SUBJECTIVE
Correct Marks : 3

Solve the PDE $2p + 3q = 1$ using Lagrange's method.

Question Number : 22 Question Id : 86435111114 Question Type : SUBJECTIVE
Correct Marks : 3

Derive general solution of the PDE $(D^2 - a^2D'^2 + 2abD + 2a^2bD')z = 0$

Question Number : 23 Question Id : 86435111115 Question Type : SUBJECTIVE
Correct Marks : 3

Find a particular integral of the equation

$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x + y)$.

Question Number : 24 Question Id : 86435111116 Question Type : SUBJECTIVE
Correct Marks : 3

Obtain the general solution of $(x^2D^2 + xyDD' - 2y^2D'^2 - xD - 6yD')z = 0$

Question Number : 25 Question Id : 86435111117 Question Type : SUBJECTIVE
Correct Marks : 3

Show that the direction cosines of the tangent at a point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1,\ x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

Question Number : 26 Question Id : 86435111118 Question Type : SUBJECTIVE

Correct Marks : 3

Verify that the following equation is integrable and find its primitive.

$(6x + yz)dx + (zx - 2y)dy + (xy + 2z)dz = 0$

Question Number : 27 Question Id : 86435111119 Question Type : SUBJECTIVE
Correct Marks : 3

Use Charpit's method to solve $q + px = p^2$.

Question Number : 28 Question Id : 86435111120 Question Type : SUBJECTIVE
Correct Marks : 3

Find d'Alembert's solution of the problem

$u_{tt} - c^2 u_{xx} = 0, \quad u(x,0) = x^3, u_t(x,0) = \sin x, \quad -\infty < x < \infty, \quad t \geq 0.$

Question Number : 29 Question Id : 86435111121 Question Type : SUBJECTIVE
Correct Marks : 3

Show that for the KdV equation given by $u_t - 6uu_x + u_{xxx} = 0$, the total mass and energy are conserved, if u and its derivatives decay rapidly as $|x| \to \infty$.

Question Number : 30 Question Id : 86435111122 Question Type : SUBJECTIVE
Correct Marks : 3

Find the equation of the system of surfaces which cut orthogonally the system of cones $x^2 + y^2 + z^2 = cxy, c$ being a parameter.

Partial Differential Equations-3

Section Id :	864351550
Section Number :	3
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	7
Number of Questions to be attempted :	5
Section Marks :	50
Mark As Answered Required? :	Yes
Sub-Section Number :	1
Sub-Section Id :	864351596
Question Shuffling Allowed :	No

Question Number : 31 Question Id : 86435111123 Question Type : SUBJECTIVE
Correct Marks : 10

Classify and reduce the canonical form of the differential equation $y^2 u_{xx} - x^2 u_{yy} = 0$.

Question Number : 32 Question Id : 86435111124 Question Type : SUBJECTIVE
Correct Marks : 10

Prove that the solution of the non-homogeneous wave equation

$u_{tt} - c^2 u_{xx} = F(x,t)$ subject to the Cauchy conditions $u(x,0) = h(x)$ and

$u_t(x,0) = k(x)$ is

$$u(x,t) = \frac{1}{2}[h(x-ct) + h(x+ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} k(s)ds + \frac{1}{2c}\int_0^t \int_{x-ct+ct'}^{x+ct-ct'} F(x',t')dx'dt'$$

Question Number : 33 Question Id : 86435111125 Question Type : SUBJECTIVE
Correct Marks : 10

Find the temperature distribution at any point within the rectangle $0 \le x \le a, 0 \le y \le b$ for $t > 0,$ if the boundaries of the rectangle are maintained at zero temperature and the temperature of the slab at $t = 0$ is $Asin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$.

Question Number : 34 Question Id : 86435111126 Question Type : SUBJECTIVE
Correct Marks : 10

Use Fourier sine transform to solve the following problem.

$u_{tt} = c^2 u_{xx}, \qquad 0 \le x < \infty, t > 0$

Subject to $u(x,0) = f(x),\ u_t(x,0) = g(x),\ 0 \le x < \infty$

$u(0,t) = 0,\ t \ge 0$ and $u, u_x \to 0$ as $x \to \infty$

Question Number : 35 Question Id : 86435111127 Question Type : SUBJECTIVE
Correct Marks : 10

A homogeneous thermally conducting cylinder occupies the region ,
$0 \le r \le a, 0 \le \theta \le 2\pi, 0 \le z \le h$, where (r, θ, z) are the cylindrical coordinates. The top $z = h$ and the lateral surface $r = a$ are held at 0^o temperature, while the base $z = 0$ is held at 100^o. Assuming that there are no sources of heat generation within the cylinder, find the steady temperature distribution within the cylinder.

Question Number : 36 Question Id : 86435111128 Question Type : SUBJECTIVE
Correct Marks : 10

Find a Green's function for the heat flow problem in a finite rod described by

$$u_t = \kappa u_{xx}, \qquad 0 \le x \le L$$

Subject to $u(x,0) = f(x), 0 \le x \le L$

$$u(0,t) = u(L,t) = 0, \; t > 0.$$

Also show that the Green's function satisfies the conditions

$i) \frac{\partial G}{\partial t} = \kappa \nabla^2 G, \qquad ii) \; G(x, y, t - t') = 0$ for $y = 0, L.$

Question Number : 37 Question Id : 86435111129 Question Type : SUBJECTIVE
Correct Marks : 10

Find the complete integral of the following equations

$i) \frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz} \qquad ii) \frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$