National Testing Agency

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Partial Differential Equations

Group Number :	1
Group Id :	603489203
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Break time :	0
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Is this Group for Examiner? :	No

Partial Differential Equations-1

Section Id :	603489301
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory

Number of Questions :	20
Number of Questions to be attempted :	20
Section Marks :	20
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	603489539
Question Shuffling Allowed :	Yes

Question Number : 1 Question Id : 60348914560 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The partial differential equation $xyz_x + x^2yz_y = x^2y^2z^2$ is

1. a linear equation
2. a semi-linear equation
3. a quasi-linear equation
4. a nonlinear equation

Options :

60348954697. 1

60348954698. 2

60348954699. 3

60348954700. 4

Question Number : 2 Question Id : 60348914561 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

A mathematical problem given by $a(x,y)u_{xx} + b(x,y)u_{xy} + c(x,y)u_{yy} = \phi(x,y,u)$ together with auxiliary conditions, $u(x,0) = f(x)$, $\frac{\partial u}{\partial n}(x,0) = g(x)$ is called

1. initial value problem
2. boundary value problem
3. initial- boundary value problem
4. none of these

Options :

60348954701. 1

60348954702. 2

60348954703. 3

60348954704. 4

Question Number : 3 Question Id : 60348914562 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The function $(x-y)^n$, where n is any positive integer, is a solution of the equation

1. $u_x - u_y = 0$
2. $u_x + u_y = 0$
3. $u_x + nu_y = 0$
4. $nu_x + u_y = 0$

Options :

60348954705. 1

60348954706. 2

60348954707. 3

60348954708. 4

Question Number : 4 Question Id : 60348914563 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The general solution of $u_{tt} - c^2 u_{xx} = 0$ is given by

1. $u = g(x - ct)$
2. $u = f(x + ct)$
3. $u = f(x + ct) + g(x - ct)$
4. None of these

Options :

60348954709. 1

60348954710. 2

60348954711. 3

60348954712. 4

Question Number : 5 Question Id : 60348914564 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The first order PDE formed by eliminating arbitrary constants a and c from the equation $x^2 + y^2 + (z - c)^2 = a^2$ is given by

1. $qx - py = 0$
2. $qx + py = 0$
3. $qy + px = 0$
4. $qy - px = 0$

Options :

60348954713. 1

60348954714. 2

60348954715. 3

60348954716. 4

Question Number : 6 Question Id : 60348914565 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The general solution of $zp + x = 0$ is given by

1. $\phi(x^2 + y^2, y^2 + z^2) = 0$
2. $\phi(y^2 + z^2, x) = 0$
3. $\phi(x^2 + y^2, z) = 0$
4. $\phi(x^2 + z^2, y) = 0$

Options :

60348954717. 1

60348954718. 2

60348954719. 3

60348954720. 4

Question Number : 7 Question Id : 60348914566 Question Type : MCQ Option Shuffling : No Is

Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The primitive of the Pfaffian differential equation

$\frac{dx}{x}+\frac{y^2}{z(y+z)}d\left(\frac{z}{y}\right)=0$ is given by

1. $yz = c(x + z)$
2. $xy = c(y + z)$
3. $xz = c(y + z)$
4. $z = c(x + y)$

Options :

60348954721. 1

60348954722. 2

60348954723. 3

60348954724. 4

Question Number : 8 Question Id : 60348914567 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The complete integral of $p = e^q$ is given by

1. $z = e^a x + by + a$
2. $z = e^a x + ay + b$
3. $z = ax + e^a y + b$
4. $z = ax + by + a$

Options :

60348954725. 1

60348954726. 2

60348954727. 3

60348954728. 4

Question Number : 9 Question Id : 60348914568 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The general solution of $(D+1)(D+D'-1)z=0$ is given by

1. $z=e^{-x}\phi_1(y)+e^{x}\phi_2(y-x)$
2. $z=e^{x}\phi_1(y)+e^{-x}\phi_2(y-x)$
3. $z=e^{-x}\phi_1(x)+e^{x}\phi_2(y-x)$
4. $z=e^{-x}\phi_1(x)+e^{-x}\phi_2(y-x)$

Options :

60348954729. 1

60348954730. 2

60348954731. 3

60348954732. 4

Question Number : 10 Question Id : 60348914569 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

A particular integral of $(2DD'+D'^2)z=3\cos(3x-2y)$ is

1. $z=\frac{3}{8}\cos(3x-2y)$
2. $z=\frac{3}{8}\sin(3x-2y)$
3. $z=\frac{1}{8}\cos(3x-2y)$
4. $z=\frac{1}{8}\sin(3x-2y)$

Options :

60348954733. 1

60348954734. 2

60348954735. 3

60348954736. 4

Question Number : 11 Question Id : 60348914570 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The PDE $e^x z_{xx} + e^y z_{yy} = z$ is

1. hyperbolic $\forall x \in R \text{ and } y \in R$
2. elliptic $\forall x \in R \text{ and } y \in R$
3. parabolic $\forall x \in R \text{ and } y \in R$
4. none of these

Options :

60348954737. 1

60348954738. 2

60348954739. 3

60348954740. 4

Question Number : 12 Question Id : 60348914571 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The two characteristic directions for the PDE

$u_{xx} + x u_{yy} = 0$ for $x > 0$ are given by

1. $\frac{dy}{dx} = 1 + x, \quad \frac{dy}{dx} = -(1 + x)$
2. $\frac{dy}{dx} = i\sqrt{1 + x}, \quad \frac{dy}{dx} = -i\sqrt{1 + x}$
3. $\frac{dy}{dx} = i\sqrt{x}, \frac{dy}{dx} = -i\sqrt{x}$
4. $\frac{dy}{dx} = \sqrt{x}, \frac{dy}{dx} = -\sqrt{x}$

Options :

60348954741. 1

60348954742. 2

60348954743. 3

60348954744. 4

Question Number : 13 Question Id : 60348914572 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The PDE $(x^3y^2D^3D'^2 - x^2y^3D^2D'^3)u = 0$ is given a transformation , $x = e^z, y = e^t$. The changed equation is_____, Where , $D_1 \equiv \frac{d}{dz}$, and $D_1' \equiv \frac{d}{dt}$

1. $D_1D_1'(D_1 - D_1')(D_1 - 2D_1')(D_1 - 3D_1')u = 0$
2. $(D_1^3D_1'^2 - D_1^2D_1'^3)u = 0$
3. $D_1D_1'(D_1 - 1)(D_1' - 1)(D_1 - D_1')u = 0$
4. None of these

Options :

60348954745. 1

60348954746. 2

60348954747. 3

60348954748. 4

Question Number : 14 Question Id : 60348914573 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

Riemann's method is generally used to find solution of

1. a nonlinear parabolic equation
2. linear parabolic equation
3. nonlinear hyperbolic equation
4. linear hyperbolic equation

Options :

60348954749. 1

60348954750. 2

60348954751. 3

60348954752. 4

Question Number : 15 Question Id : 60348914574 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The solution of initial value problem $u_{tt} - c^2 u_{xx} = x$ subject to $u(x,0) = 0,\ u_t(x,0) = 3$ is

1. $u(x,t) = 3t + \frac{1}{2}xt^2$
2. $u(x,t) = 3t - \frac{1}{2}xt^2$
3. $u(x,t) = 3e^t + \frac{1}{2}xt^2$
4. None of these

Options :

60348954753. 1

60348954754. 2

60348954755. 3

60348954756. 4

Question Number : 16 Question Id : 60348914575 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The solution of Laplace equation $u_{xx} + u_{yy} = 0, 0 \le x \le 1, 0 \le y \le 1$ together with boundary conditions $u = 1$ on the sides $x = 0,1,\ y = 0,1$ is

1. $u = xy$
2. $u = 1$
3. $u = x(x-1)y(y-1)$
4. none of these

Options :

60348954757. 1

60348954758. 2

60348954759. 3

60348954760. 4

Question Number : 17 Question Id : 60348914576 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The Fourier transform of $\frac{\sin(ax)}{x}$ is

1. 0
2. $\frac{\sqrt{\pi}}{2}$
3. $\sqrt{\frac{\pi}{2}}\, H(a-|p|)$
4. None of these

Options :

60348954761. 1

60348954762. 2

60348954763. 3

60348954764. 4

Question Number : 18 Question Id : 60348914577 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The Laplace transform of $\sin(ax)$ is

1. $\frac{p}{p^2-a^2}$
2. $\frac{a}{p^2-a^2}$
3. $\frac{p}{p^2+a^2}$
4. $\frac{a}{p^2+a^2}$

Options :

60348954765. 1

60348954766. 2

60348954767. 3

60348954768. 4

Question Number : 19 Question Id : 60348914578 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

The equation $u_t + c(u)u_x = 0$ is known as

1. nonlinear wave equation
2. linear wave equation
3. Burger's equation
4. KdV equation

Options :

60348954769. 1

60348954770. 2

60348954771. 3

60348954772. 4

Question Number : 20 Question Id : 60348914579 Question Type : MCQ Option Shuffling : No Is Question Mandatory : No

Correct Marks : 1 Wrong Marks : 0

If the Fourier transform of $f_1(x)$ is $F_1(p)$ and $f_2(x)$ is $F_2(p)$ then Fourier transform of

$\int_0^x f_1(x-x')f_2(x')dx'$ is

1. $F_1(p) + F_2(p)$
2. $F_1(p) - F_2(p)$
3. $F_1(p)/F_2(p)$
4. $F_1(p)F_2(p)$

Options :

60348954773. 1

60348954774. 2

60348954775. 3

60348954776. 4

Partial Differential Equations-2

Section Id :	603489302
Section Number :	2
Section type :	Offline

Mandatory or Optional :	Mandatory
Number of Questions :	10
Number of Questions to be attempted :	10
Section Marks :	30
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	603489540
Question Shuffling Allowed :	No

Question Number : 21 Question Id : 60348914580 Question Type : SUBJECTIVE

Correct Marks : 3

Solve the PDE $ptan(x) + qtan(y) = \tan(z)$ using Lagrange's method

Question Number : 22 Question Id : 60348914581 Question Type : SUBJECTIVE

Correct Marks : 3

Derive general solution of the PDE $(D^2 + 2DD' + D'^2 + 2D + 2D' + 1)z = 0.$

Question Number : 23 Question Id : 60348914582 Question Type : SUBJECTIVE

Correct Marks : 3

Find a particular integral of the equation

$(D - 1)(D' + 1)z = xy.$

Question Number : 24 Question Id : 60348914583 Question Type : SUBJECTIVE

Correct Marks : 3

Obtain the general solution of $(x^2D^2 - 4y^2D'^2 - 4yD' - 1)z = 0.$

Question Number : 25 Question Id : 60348914584 Question Type : SUBJECTIVE

Correct Marks : 3

Find the condition such that the plane $lx + my + nz + p = 0$ should touch the central conicoid $ax^2 + by^2 + cz^2 = 1$.

Question Number : 26 Question Id : 60348914585 Question Type : SUBJECTIVE

Correct Marks : 3

Prove that the necessary condition for existence of a relation of the form $F(u, v) = 0$ between two functions $u(x, y)$ and $v(x, y)$, not involving x or y explicitly is that

$\frac{\partial(u, v)}{\partial(x, y)} = 0.$

Question Number : 27 Question Id : 60348914586 Question Type : SUBJECTIVE

Correct Marks : 3

Use Charpit's method to solve $(q^2 + p^2)x = pz$.

Question Number : 28 Question Id : 60348914587 Question Type : SUBJECTIVE

Correct Marks : 3

Find d'Alembert's solution to determine the deflection of a vibrating string of unit length having fixed ends with zero initial velocity and initial deflection $f(x) = a \sin^2 \pi x$, c being the wave speed.

Question Number : 29 Question Id : 60348914588 Question Type : SUBJECTIVE

Correct Marks : 3

For the nonlinear Schrodinger equation given by

$i\psi_t + \psi_{xx} + \gamma|\psi|^2\psi = 0, \ -\infty < x < \infty, \ t \geq 0$, prove that

$\int_{-\infty}^{\infty} |\psi|^2 dx = constant$, where $|\psi| \to 0$ as $|x| \to \infty$.

Question Number : 30 Question Id : 60348914589 Question Type : SUBJECTIVE

Correct Marks : 3

Show that the equations $px - qy = x$ and $x^2p + q = xz$ are compatible.

Partial Differential Equations-3

Section Id :	603489303
Section Number :	3
Section type :	Offline
Mandatory or Optional :	Mandatory
Number of Questions :	7
Number of Questions to be attempted :	5
Section Marks :	50
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Sub-Section Number :	1
Sub-Section Id :	603489541
Question Shuffling Allowed :	No

Question Number : 31 Question Id : 60348914590 Question Type : SUBJECTIVE

Correct Marks : 10

Classify and reduce the canonical form of the differential equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0.$

Question Number : 32 Question Id : 60348914591 Question Type : SUBJECTIVE

Correct Marks : 10

Use Riemann's method to find solution of the initial value problem

$u_{tt} = c^2 u_{xx}, -\infty < x < \infty, t \geq 0,$ subject to

$u(x,0) = \eta(x)$ and $\frac{\partial u}{\partial t}(x,0) = v(x).$

Question Number : 33 Question Id : 60348914592 Question Type : SUBJECTIVE

Correct Marks : 10

Find the solution of $\frac{\partial T}{\partial t} = \kappa\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2\partial T}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial T}{\partial \theta}\right)\right)$ using the method of separation of variables.

Question Number : 34 Question Id : 60348914593 Question Type : SUBJECTIVE

Correct Marks : 10

Solve using Hankel transformation , $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial t^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right),\ 0 \leq r < \infty, t > 0$ with initial conditions $u(r,0) = f(r)$ and $\frac{\partial u}{\partial t}(r,0) = g(r).$

Question Number : 35 Question Id : 60348914594 Question Type : SUBJECTIVE

Correct Marks : 10

Find the potential function $\psi(x,y,z)$ in a rectangular box such that $\nabla^2\psi = 0$ in $0 \leq x \leq a,\ 0 \leq y \leq b,\ 0 \leq z \leq c,$ if the potential is zero on all sides and the bottom, while the potential $\psi = f(x,y)$ on the top of the box.

Question Number : 36 Question Id : 60348914595 Question Type : SUBJECTIVE

Correct Marks : 10

Prove that the Green's function $G(\boldsymbol{r}, \boldsymbol{\xi})$ is symmetric.

Question Number : 37 Question Id : 60348914596 Question Type : SUBJECTIVE

Correct Marks : 10

a) Find the equation of the system of surfaces which cut orthogonally the system of cones , $x^2 + y^2 + z^2 = cxy, c$ being of a parameter.

b) Write fundamental solution of Laplace equation in 2D and 3D. Verify your answer.