

JNUEE MSC Mathematics

1) Which of the following can be the class equation of a group of order 12 ?[Question ID = 34921][Question Description = M.Sc.MATM_Q_001]

1. $12=1+1+1+2+7$ [Option ID = 208945]
2. $12=1+1+2+2+3+3$ [Option ID = 208946]
3. $12=1+1+1+1+1+3+4$ [Option ID = 208947]
4. $12=1+1+1+1+3+5$ [Option ID = 208948]

2) Let G be a group of order 7 . Which of the following statements is/are correct?

- A. There is an injective homomorphism from G into S_7 .
- B. There is an injective homomorphism from G into $GL_2(\mathbb{R})$.
- C. There is an injective homomorphism from G into $SL_7(\mathbb{Z})$.

[Question ID = 34922][Question Description = M.Sc.MATM_Q_002]

1. A only
[Option ID = 208949]
2. B only
[Option ID = 208950]
3. A and B only
[Option ID = 208951]
4. All statements of A, B, C are true
[Option ID = 208952]

3) Exactly how many group homomorphisms are there from S_4 to $\mathbb{Z}/24\mathbb{Z}$?[Question ID = 34923][Question Description = M.Sc.MATM_Q_003]

1. 1 [Option ID = 208953]
2. 2 [Option ID = 208954]
3. ${}^{12}C_2$ [Option ID = 208955]
4. 4 [Option ID = 208956]

4) Let A be a 4×4 matrix with real entries. Suppose $i = \sqrt{-1} \in \mathbb{C}$. Suppose that the characteristic values of A are $1, -1, i, -i$. Then we have

[Question ID = 34924][Question Description = M.Sc.MATM_Q_004]

1. $A^{-1}=A$
[Option ID = 208957]
2. $A^{-1}=A^2$
[Option ID = 208958]
3. $A^{-1}=A^3$
[Option ID = 208959]
4. $A^{-1}=A^4$
[Option ID = 208960]

5) For $n \geq 2$, let U_n be the group of invertible elements in $\mathbb{Z}/n\mathbb{Z}$ with respect to multiplication modulo n . Then choose the correct option.

[Question ID = 34925][Question Description = M.Sc.MATM_Q_005]

1. the group U_6 is cyclic, but U_8 is not cyclic
[Option ID = 208961]
2. the group U_8 is cyclic, but U_6 is not cyclic
[Option ID = 208962]
3. Both the groups U_6 and U_8 are cyclic
[Option ID = 208963]
4. Neither the group U_6 nor U_8 is cyclic
[Option ID = 208964]

6) Let V_1 and V_2 be two dimensional real inner product spaces whose underlying vector space is $\mathbb{R}^2$. Which of the following statements is/are correct?

- A. A circle in V_1 is a circle in V_2 .
- B. A square in V_1 is a square in V_2 .
- C. A parallelogram in V_1 is a parallelogram in V_2 .

[Question ID = 34926][Question Description = M.Sc.MATM_Q_006]

1. A only
[Option ID = 208965]
2. B and C only
[Option ID = 208966]
3. C only

[Option ID = 208967]

4. All of A, B and C

[Option ID = 208968]

- 7) Let $S = \{v_1, v_2, \dots, v_{n-1}\}$ be an orthonormal subset of $\mathbb{R}^n$ with standard inner product. Then the number of vectors $v_0 \in \mathbb{R}^n$ such that $S \cup \{v_0\}$ is an orthonormal basis of $\mathbb{R}^n$ is

[Question ID = 34927][Question Description = M.Sc.MATM_Q_007]

1. 1

[Option ID = 208969]

2. 2

[Option ID = 208970]

3. n

[Option ID = 208971]

4. infinite

[Option ID = 208972]

- 8) In $\mathbb{R}^2$ consider any two vectors $v = (x_1, y_1), w = (x_2, y_2)$. The following functions are candidates for an inner product. Determine which of them define inner products on $\mathbb{R}^2$?

A. $f(v, w) = x_1x_2 + y_1y_2 + x_1 + x_2$

B. $g(v, w) = 3x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1$

C. $h(v, w) = x_1x_2 - y_1y_2 + x_1y_2 + x_2y_1$

[Question ID = 34928][Question Description = M.Sc.MATM_Q_008]

1. None of f, g and h

[Option ID = 208973]

2. f and g only

[Option ID = 208974]

3. h only

[Option ID = 208975]

4. g only

[Option ID = 208976]

9) Which of the following sets has/have the same cardinality as $\mathbb{R}$?

A. The set of all real sequences

B. The set of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$

C. The set of all functions from $\mathbb{R}$ to the two-element set $\{0, 1\}$

Choose the *correct* answer from the options given below: [Question ID = 34929][Question Description = M.Sc.MATM_Q_009]

1. A only [Option ID = 208977]
2. B only [Option ID = 208978]
3. A and B only [Option ID = 208979]
4. C only [Option ID = 208980]

10)

$\lim_{x \rightarrow 0^+} \frac{x^x - 1}{x}$ is equal to

[Question ID = 34930][Question Description = M.Sc.MATM_Q_010]

1. 0 [Option ID = 208981]
2. -1 [Option ID = 208982]
3. -e [Option ID = 208983]
4. $-\infty$ [Option ID = 208984]

11) In how many ways can we choose the sets A, B, C so that $A \cup B \cup C = \{1, 2, 3, 4\}$? [Question ID = 34519][Question Description = M.Sc.MATM_Q_011]

1. 7^4 [Option ID = 208985]
2. 3^4 [Option ID = 208986]
3. 4^3 [Option ID = 208987]
4. $3!4!$ [Option ID = 208988]

12) What is the highest power of 2 dividing the product $(2020)(2021)(2022)\dots(4040)$? [Question ID = 34520][Question Description = M.Sc.MATM_Q_012]

1. 2^{2022} [Option ID = 208989]
2. 2^{2020} [Option ID = 208990]
3. 2^{1011} [Option ID = 208991]
4. 2^{1518} [Option ID = 208992]

13) Three numbers are chosen randomly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability that their product is divisible by 10? [Question ID = 34521][Question Description = M.Sc.MATM_Q_013]

1. 8/15 [Option ID = 208993]
2. 29/60 [Option ID = 208994]
3. 31/60 [Option ID = 208995]
4. 1/2 [Option ID = 208996]

14) $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{100}}$ is

[Question ID = 34522][Question Description = M.Sc.MATM_Q_014]

1. between 17 and 18 [Option ID = 208997]
2. between 18 and 19 [Option ID = 208998]
3. between 19 and 20 [Option ID = 208999]
4. between 20 and 21 [Option ID = 209000]

15) 100 women participate in a knockout tournament. (In a knockout tournament a player is out of the tournament as soon as she loses one match. Some players might get byes in the first round.) How many matches will be held in the tournament?[Question ID = 34523][Question Description = M.Sc.MATM_Q_015]

1. $\left\lfloor \frac{100}{2} \right\rfloor + \left\lfloor \frac{100}{2^2} \right\rfloor + \dots + \left\lfloor \frac{100}{2^6} \right\rfloor$

[Option ID = 209001]

2. 98 [Option ID = 209002]
3. 99 [Option ID = 209003]
4. 100 [Option ID = 209004]

16) How many subsets of $\{1, 2, 3, \dots, 50\}$ have 20 elements but contain no two consecutive natural numbers in them?[Question ID = 34524][Question Description = M.Sc.MATM_Q_016]

1. $\binom{50}{19}$

[Option ID = 209005]

2. $\binom{30}{20}$

[Option ID = 209006]

3. $\binom{31}{20}$

[Option ID = 209007]

4. $\binom{30}{19}$

[Option ID = 209008]

17) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(nx) = 2nf(x)$ for all $n \in \mathbb{N}$ and all $x \in \mathbb{R}$. Consider the following statements:

A. f is continuous.

B. f is a group homomorphism.

Which of the following is correct?

[Question ID = 34525][Question Description = M.Sc.MATM_Q_017]

1. A is true but B is false.

[Option ID = 209009]

2. B is true but A is false.

[Option ID = 209010]

3. Both A and B are true.

[Option ID = 209011]

4. Both A and B are false.

[Option ID = 209012]

18) Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a continuous function such that $f(\alpha v) = f(v)$ for all $\alpha \in \mathbb{R}, \alpha > 0$ and all $v \in \mathbb{R}$. Consider the following statements:

A. The image of f is compact.

B. The image of f is uncountable.

Which of the following is correct?

[Question ID = 34526][Question Description = M.Sc.MATM_Q_018]

1. A is true but B is false.
[Option ID = 209013]
2. B is true but A is false.
[Option ID = 209014]
3. Both A and B are true.
[Option ID = 209015]
4. Both A and B are false.
[Option ID = 209016]

- 19) Let $\{C_n\}_{n \in \mathbb{N}}$ be a sequence of nonempty connected subsets of $\mathbb{R}^2$ such that $C_{n+1} \subset C_n, n \in \mathbb{N}$. Consider the following statements:
- A. $\bigcap_{n=1}^{\infty} C_n$ is nonempty.
- B. $\bigcap_{n=1}^{\infty} C_n$ is connected.
- Which of the following is necessarily correct?

[Question ID = 34527][Question Description = M.Sc.MATM_Q_019]

1. A is true but B is false. [Option ID = 209017]
2. B is true but A is false. [Option ID = 209018]
3. Both A and B are true. [Option ID = 209019]
4. Both A and B are false. [Option ID = 209020]

- 20) Let $\{a_n\}_{n \in \mathbb{N}} \subset \mathbb{R}^2$ be such that $a_n = (\cos n, \sin n), n \in \mathbb{N}$. Which of the following statements is correct?

[Question ID = 34528][Question Description = M.Sc.MATM_Q_020]

1. $\{a_n\}_{n \in \mathbb{N}}$ is convergent.
[Option ID = 209021]
2. $\{a_n\}_{n \in \mathbb{N}}$ is not convergent but it has a convergent subsequence.
[Option ID = 209022]
3. $\{a_n\}_{n \in \mathbb{N}}$ has no convergent subsequence.
[Option ID = 209023]
4. $\{a_n\}_{n \in \mathbb{N}}$ is unbounded.
[Option ID = 209024]

- 21) What is the value of $\lim_{x \rightarrow 0} \frac{|x|}{\sin^2 x}$?

[Question ID = 34529][Question Description = M.Sc.MATM_Q_021]

1. 1
[Option ID = 209025]
2. 0
[Option ID = 209026]
3. ∞
[Option ID = 209027]
4. π
[Option ID = 209028]

- 22) For any pair of subsets C and D of $\mathbb{R}^2$, let C^0 denote the interior of C , and $C \setminus D$ denote the set of points in C which do not belong to D . Consider the following statements:
- A. $(C \setminus D)^0 = C^0 \setminus D^0$
- B. $(C^0 \setminus D)^0 = C^0 \setminus D^0$
- Which of the following is necessarily correct?

[Question ID = 34530][Question Description = M.Sc.MATM_Q_022]

1. A is true but B is false.
[Option ID = 209029]
2. B is true but A is false.
[Option ID = 209030]
3. Both A and B are true.
[Option ID = 209031]
4. Both A and B are false.
[Option ID = 209032]

23) Sports Class: Please answer the question based on the information given below.

In a class of 60 students, 35 students enrolled for football, 45 students enrolled for basketball 55 and students enrolled for hockey.

What is minimum number of students who must have enrolled for at least two sports?

[Question ID = 34607][Question Description = M.Sc.MATM_Q_023]

1. 0
[Option ID = 209337]
2. 20
[Option ID = 209338]
3. 40
[Option ID = 209339]
4. 45
[Option ID = 209340]

24) Sports Class: Please answer the question based on the information given below.

In a class of 60 students, 35 students enrolled for football, 45 students enrolled for basketball 55 and students enrolled for hockey.

What is the minimum number of students who must have enrolled for all three sports?

[Question ID = 34608][Question Description = M.Sc.MATM_Q_024]

1. 0
[Option ID = 209341]
2. 15
[Option ID = 209342]
3. 25
[Option ID = 209343]
4. 35
[Option ID = 209344]

25)

Consider the subset $S := \{(x, y) \in \mathbb{R}^2 : y = x \text{ or } |x| + |y| = 1\}$ of $\mathbb{R}^2$.

Consider the following assertions about S:

- A. S is a reflexive relation on $\mathbb{R}$.
- B. S is a transitive relation on $\mathbb{R}$.
- C. There exists a surjective map from $\mathbb{R}$ onto S.
- D. There exists an injective map from S into $\mathbb{Q}^3$.

Which of the above assertions is/are correct?

[Question ID = 34531][Question Description = M.Sc.MATM_Q_025]

1. A only [Option ID = 209033]
2. A and D only [Option ID = 209034]
3. A and C only [Option ID = 209035]
4. A and B only [Option ID = 209036]

26)

Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\} \subset \mathbb{R}$.

Which of the following assertions about S is correct?

[Question ID = 34532][Question Description = M.Sc.MATM_Q_026]

1. For any two maps $f : \mathbb{Q} \times \mathbb{Q} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow S$, their composition $g \circ f$ is never surjective.
[Option ID = 209037]
2. There exists a surjective map from S onto $\mathbb{R}^2/\mathbb{Z}^2$ [Option ID = 209038]
3. There exists an injective map from $\mathbb{R}/\mathbb{Q}$ into S. [Option ID = 209039]
4. For any two maps $f : S \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}^3$, their composition $g \circ f$ has a

[Option ID = 209040]

27)

Consider the following assertions:

A. For each $0 < y < 1$, there exists a $0 < x < y$ such that $e^y = 1 + ye^x$.B. $|1 - \cos(x)| \leq x$ for all $x > 0$.C. $|\sin(x) - \sin(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

Which of the above assertions is/are correct?

[Question ID = 34533][Question Description = M.Sc.MATM_Q_027]

1. B and C only [Option ID = 209041]
2. C only [Option ID = 209042]
3. All of A, B and C are correct. [Option ID = 209043]
4. A and C only [Option ID = 209044]

28)

Consider the following assertions:

A. There exists an $x < 0$ such that $e^{\sin(2x)} = \frac{1}{2}$.B. There exists an $x > 0$ such that $e^{\cos(2x)} = \frac{1}{6}$.C. There exists an $x \in \mathbb{R}$ such that $e^{|\sin(2x)|} = 2$.

Which of the above assertions is/are true?

[Question ID = 34534][Question Description = M.Sc.MATM_Q_028]

1. All of A, B and C are true. [Option ID = 209045]
2. A and C only [Option ID = 209046]
3. B and C only [Option ID = 209047]
4. A and B only [Option ID = 209048]

29)

Consider the rectangle $D = [0, 1] \times [0, 2]$ and the continuous vector field $\vec{F} : D \rightarrow \mathbb{R}^2$ given by $\vec{F}(x, y) = -y\hat{i} + x\hat{j}$. What is the line integral of $\vec{F}$ along the curve C givenby $C : y = 2x^2, 0 \leq x \leq 1$ in D ?

[Question ID = 34535][Question Description = M.Sc.MATM_Q_029]

1. 0 [Option ID = 209049]
2. $1/3$ [Option ID = 209050]
3. $2/3$ [Option ID = 209051]
4. $4/3$ [Option ID = 209052]

30)

Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = x^2 + \pi$. Which of the following assertions about f is/are true?

[Question ID = 34536][Question Description = M.Sc.MATM_Q_030]

1. A and D only [Option ID = 209053]
2. B and D only [Option ID = 209054]
3. B and C only [Option ID = 209055]
4. B only [Option ID = 209056]

31)

Let $f : [-1, 1] \rightarrow \mathbb{R}$ be a non constant function which is continuous at 0 with $f(0) = 1$.

Consider the following statements:

A. There exists $\epsilon > 0$ such that $f(x) > \frac{2020}{2021}$ for all $x \in]0, \epsilon[$.B. There exists $\epsilon > 0$ such that $f(x) < \frac{2020}{2021}$ for all $x \in]-\epsilon, 0[$.

Then the correct statement(s) is/are:

[Question ID = 34537][Question Description = M.Sc.MATM_Q_031]

1. A is true and B is false. [Option ID = 209057]
2. B is true and A is false. [Option ID = 209058]
3. both A and B are true. [Option ID = 209059]
4. both A and B are false. [Option ID = 209060]

32)

Let $f : [-2021, 2021] \rightarrow \mathbb{R}$ be a function such that $|f(x) - f(y)| \leq 2021|x - y|$ for all $x, y \in [-2021, 2021]$ and further $f(20) = 21$ and $f(21) = 20$. Consider the following statements:

- A. f is a continuous function.
- B. There is an x with $-41 < x < 41$ such that $f(x) = x$.

Then

[Question ID = 34538][Question Description = M.Sc.MATM_Q_032]

1. both A and B are true. [Option ID = 209061]
2. B is true and A is false. [Option ID = 209062]
3. both A and B are false. [Option ID = 209063]
4. A is true and B is false. [Option ID = 209064]

33)

Let $f :]0, \infty[\rightarrow \mathbb{R}$ be defined as $f(x) = \frac{xe^{-x}}{1+x}$. Then

[Question ID = 34539][Question Description = M.Sc.MATM_Q_033]

1. f is a strictly increasing function on $]0, \infty[$.
[Option ID = 209065]
2. f is a strictly decreasing function on $]0, \infty[$.
[Option ID = 209066]
3. there exists $x_0 \in]0, \infty[$ such that $f(x_0)f''(x_0) < 0$.
[Option ID = 209067]
4. f is an onto function [Option ID = 209068]

34)

For each $\mathbf{a} = (a_0, a_1, a_2, a_3) \in \mathbb{R}^4$, let $f_{\mathbf{a}} : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f_{\mathbf{a}}(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3.$$

Consider the following statements:

- A. $f_{\mathbf{a}}$ is continuous at $x = 0$ for any $\mathbf{a} \in \mathbb{R}^4$.
- B. $f_{\mathbf{a}}$ is differentiable at $x = 0$ for any $\mathbf{a} \in \mathbb{R}^4$.

Then

[Question ID = 34540][Question Description = M.Sc.MATM_Q_034]

1. A is true and B is false.
[Option ID = 209069]
2. both A and B are false.
[Option ID = 209070]
3. both A and B are true.
[Option ID = 209071]
4. B is true and A is false.
[Option ID = 209072]

35)

The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^5 \log_e n}{4^n} x^n$$

is

[Question ID = 34541][Question Description = M.Sc.MATM_Q_035]

1. 0 [Option ID = 209073]
2. 4 [Option ID = 209074]
3. 5 [Option ID = 209075]
4. ∞ [Option ID = 209076]

36)

The value of $\lim_{x \rightarrow \infty} [(x^{10} - x^9)^{\frac{1}{10}} - (x^8 - x^7)^{\frac{1}{8}}]$ is

[Question ID = 34542][Question Description = M.Sc.MATM_Q_036]

1. 0 [Option ID = 209077]
2. $1/8$ [Option ID = 209078]
3. $1/10$ [Option ID = 209079]
4. $1/40$ [Option ID = 209080]

37)

Suppose $f:]-\frac{\pi}{2}, \frac{\pi}{2}[\rightarrow \mathbb{R}$ is a continuous function such that $f(x) = \frac{\sqrt{9-x} - \sqrt{9+x}}{\sin x \cos x}$ for all $x \neq 0$. Then the value of $f(0)$ is

[Question ID = 34543][Question Description = M.Sc.MATM_Q_037]

1. $-1/9$ [Option ID = 209081]
2. $-1/6$ [Option ID = 209082]
3. $-1/3$ [Option ID = 209083]
4. 0 [Option ID = 209084]

38) Consider the following statements:

A. Union of any collection of open sets in $\mathbb{R}^n$ is open.

B. Intersection of any collection of open sets in $\mathbb{R}^n$ is open. Then [Question ID = 34544][Question Description = M.Sc.MATM_Q_038]

1. B is true and A is false. [Option ID = 209085]
2. A is true and B is false. [Option ID = 209086]
3. both A and B are true. [Option ID = 209087]
4. both A and B are false. [Option ID = 209088]

39)

Suppose $\mathcal{R}$ is a relation defined on $\mathbb{N}_{\text{by}}$ $(a, b) \in \mathcal{R}$ iff $a - b$ is a multiple of 4 or a multiple of 6. Consider the statements:

A. $\mathcal{R}$ is symmetric.

B. $\mathcal{R}$ is transitive.

C. There is an infinite subset $U \subset \mathbb{N}$ of $\mathbb{N}$ such that $\mathcal{R}$ defined on U is an equivalence relation.

Then the number of correct statements above is

[Question ID = 34545][Question Description = M.Sc.MATM_Q_039]

1. 1 [Option ID = 209089]
2. 2 [Option ID = 209090]
3. 3 [Option ID = 209091]
4. 0 [Option ID = 209092]

40)

Let $s(n)$ be the sum of decimal digits of a positive integer n . Then the number of solutions of the equation $n + s(n) + s(s(n)) = 2021$ in positive integers n is

[Question ID = 34546][Question Description = M.Sc.MATM_Q_040]

1. 0 [Option ID = 209093]
2. 1 [Option ID = 209094]
3. 3 [Option ID = 209095]
4. 21 [Option ID = 209096]

41)

The number of solutions in positive integers (x, y, z) ,

with $\gcd(x, y) = \gcd(y, z) = \gcd(z, x) = 1$, of the equation

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 4 \text{ is}$$

[Question ID = 34547][Question Description = M.Sc.MATM_Q_041]

1. 0 [Option ID = 209097]
2. 1 [Option ID = 209098]
3. 2 [Option ID = 209099]
4. 4 [Option ID = 209100]

42) Given that 100003 is a prime, the remainder when 100001! is divided by 100003, is [Question ID = 34548][Question Description = M.Sc.MATM_Q_042]

1. 100002 [Option ID = 209101]
2. 0 [Option ID = 209102]
3. 1 [Option ID = 209103]
4. 2 [Option ID = 209104]

43)

Let $f(x, y) = (2xy)/(x^4 + y^2)$ when $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Then $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

[Question ID = 34549][Question Description = M.Sc.MATM_Q_043]

1. is equal to 0 [Option ID = 209105]
2. is equal to 1 [Option ID = 209106]
3. does not exist. [Option ID = 209107]
4. is equal to ∞ [Option ID = 209108]

44)

Let $f(x, y) = (xy)/(x^2 + y^2)$ when $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

Which of the following statements is true?

[Question ID = 34550][Question Description = M.Sc.MATM_Q_044]

1. f is continuous everywhere on the Euclidean plane $\mathbb{R}^2$ apart from $(0, 0)$.
[Option ID = 209109]
2. f is continuous everywhere on the Euclidean plane $\mathbb{R}^2$ apart from $(0, 0)$ but differentiable nowhere.
[Option ID = 209110]
3. f is differentiable everywhere on the Euclidean plane $\mathbb{R}^2$.
[Option ID = 209111]
4. f is continuous everywhere on the Euclidean plane $\mathbb{R}^2$ and also differentiable everywhere apart from $(0, 0)$.
[Option ID = 209112]

45)

Let $f(x, y) = x \sin(xy) + x^2 + xy + 3$. Compute the partial derivatives of f at $(0, 1)$.

[Question ID = 34551][Question Description = M.Sc.MATM_Q_045]

1. $\frac{\partial f}{\partial x}|_{(0,1)} = 0$ and $\frac{\partial f}{\partial y}|_{(0,1)} = 0$

[Option ID = 209113]

2. $\frac{\partial f}{\partial x}|_{(0,1)} = 1$ and $\frac{\partial f}{\partial y}|_{(0,1)} = 1$

[Option ID = 209114]

3. $\frac{\partial f}{\partial x}|_{(0,1)} = 0$ and $\frac{\partial f}{\partial y}|_{(0,1)} = 1$

[Option ID = 209115]

4. $\frac{\partial f}{\partial x}|_{(0,1)} = 1$ and $\frac{\partial f}{\partial y}|_{(0,1)} = 0$

[Option ID = 209116]

46)

Compute $\frac{dw}{dt}|_{t=\frac{\pi}{4}}$ where $w = xy^2z$, $x = \cos(t)$, $y = \sin(t)$, $z = t$.

[Question ID = 34552][Question Description = M.Sc.MATM_Q_046]

1. 1 [Option ID = 209117]

2. $(\pi + 4)/(8\sqrt{2})$

[Option ID = 209118]

3. $1/(2\sqrt{2})$

[Option ID = 209119]

4. $(\pi + 1)/(2\sqrt{2})$

[Option ID = 209120]

47)

Find the absolute maximum and absolute minimum of $f(x, y) = x^2 + y^2$ on the region defined by $x \geq 0, y \geq 0$ and $2x + y \leq 1$.

[Question ID = 34553][Question Description = M.Sc.MATM_Q_047]

1. Absolute maximum is 2, absolute minimum is 0 . [Option ID = 209121]
2. Absolute maximum is 1 , absolute minimum is 0. [Option ID = 209122]
3. Absolute maximum is 1/5 , absolute minimum is 0 . [Option ID = 209123]
4. Absolute maximum is 1 , absolute minimum does not exist. [Option ID = 209124]

48)

Find $\frac{df}{dx}$ where $f(x) = \int_0^{3x^2} t \sin(t) dt$.

[Question ID = 34554][Question Description = M.Sc.MATM_Q_048]

1. $3x^2 \sin 3x^2$

[Option ID = 209125]

2. $3x^2 \cos 3x^2$

[Option ID = 209126]

3. $12x^3 \sin 3x^2$

[Option ID = 209127]

4. $18x^3 \sin 3x^2$

[Option ID = 209128]

49)

Evaluate $\int_0^1 \int_0^1 2x^3 e^{x^2 y} dx dy$.

[Question ID = 34555][Question Description = M.Sc.MATM_Q_049]

1. $e^2 - e - 2$

[Option ID = 209129]

2. $e^2 - 2$

[Option ID = 209130]

3. $e - 2$

[Option ID = 209131]

4. 0 [Option ID = 209132]

50)

Find the area of the region enclosed by the curve $y = x^3$ and the line $x = y$.

[Question ID = 34556][Question Description = M.Sc.MATM_Q_050]

1. 1/2 [Option ID = 209133]
2. 1/8 [Option ID = 209134]
3. 1/16 [Option ID = 209135]
4. 1 [Option ID = 209136]

51)

Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 4$ cut out by the plane $z = 1$ and $z = 0$.

(Hint: A parametrization of the sphere is given by

$r(\theta, \phi) = (4 \sin \phi \cos \theta, 4 \sin \phi \sin \theta, 4 \cos \phi)$ where $0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi$. The

Euclidean norm of the cross product $\left\| \frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} \right\| = 4 \sin \phi$)

[Question ID = 34557][Question Description = M.Sc.MATM_Q_051]

1. 4π

[Option ID = 209137]

2. 2π

[Option ID = 209138]

3. 8π

[Option ID = 209139]

4. 16π

[Option ID = 209140]

52)

Calculate the volume of a tetrahedron whose vertices in the three dimensional Euclidean plane are $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$ and $(1, 0, 1)$.

[Question ID = 34558][Question Description = M.Sc.MATM_Q_052]

1. 1 [Option ID = 209141]

2. $1/(6\sqrt{2})$

[Option ID = 209142]

3. $1/\sqrt{2}$

[Option ID = 209143]

4. $1/6$

[Option ID = 209144]

53)

Let S be a subset of a finite dimensional vector space V over $\mathbb{R}$ such that the span of S is equal to V . Then, which of the following is necessarily true for S ?

[Question ID = 34559][Question Description = M.Sc.MATM_Q_053]

1. S is a linearly independent set. [Option ID = 209145]

2. S is a basis for V . [Option ID = 209146]

3. $|S| \geq \dim_{\mathbb{R}} V$.

[Option ID = 209147]

4. $|S| < \dim_{\mathbb{R}} V$.

[Option ID = 209148]

54)

Let $G = \mathbb{Z}/24\mathbb{Z}$ and $H = G/12G$. Then, which of the following is true?

[Question ID = 34560][Question Description = M.Sc.MATM_Q_054]

1. $H \cong \mathbb{Z}/12\mathbb{Z}$.

[Option ID = 209149]

2. $H \cong \mathbb{Z}/2\mathbb{Z}$.

[Option ID = 209150]

3. $H \cong \mathbb{Z}/24\mathbb{Z}$.

[Option ID = 209151]

4. $H \cong \mathbb{Z}/4\mathbb{Z}$.

[Option ID = 209152]

55) Let G be a group such that all quotient groups of G are cyclic. Then, which of the following is true for G ? [Question ID = 34561]
[Question Description = M.Sc.MATM_Q_055]

1. G must be a cyclic group. [Option ID = 209153]
2. G must be an abelian group, but not necessarily cyclic. [Option ID = 209154]
3. G must be a finite group. [Option ID = 209155]
4. G must be an infinite group. [Option ID = 209156]

56) Let π be a homomorphism from a group G to group H . Then, which of the following statements is correct?

[Question ID = 34562][Question Description = M.Sc.MATM_Q_056]

1. $\text{img}(\pi)$ is a subgroup of G .
[Option ID = 209157]
2. $\text{img}(\pi)$ is not a subgroup of H .
[Option ID = 209158]
3. $\text{ker}(\pi)$ is a subgroup of G .
[Option ID = 209159]
4. $\text{ker}(\pi)$ is a subgroup of H .
[Option ID = 209160]

57) Suppose G is a group and $\pi: G \rightarrow G$ is an injective homomorphism. Then, which of the following statements is necessarily true?
[Question ID = 34563][Question Description = M.Sc.MATM_Q_057]

1. G is an abelian group. [Option ID = 209161]
2. π is surjective, if $|G|$ is finite. [Option ID = 209162]
3. π is surjective, if $|G|$ is infinite. [Option ID = 209163]
4. G is a non-abelian group. [Option ID = 209164]

58) The multiplicative group of roots of unity in $\mathbb{C}^*$ is isomorphic to the group [Question ID = 34564][Question Description = M.Sc.MATM_Q_058]

1. $\mathbb{R}/\mathbb{Z}$.
[Option ID = 209165]
2. $\mathbb{Q}/\mathbb{Z}$.
[Option ID = 209166]
3. $\mathbb{C}/\mathbb{Z}$.
[Option ID = 209167]
4. $\mathbb{Z}$.
[Option ID = 209168]

59) Let V be the vector space of functions $f: \mathbb{R} \rightarrow \mathbb{R}$. Let $S = \{f \in V \mid f(1)=0, f(2)=1\}$ be a subset of V . Then, which of the following is necessarily true for S ?

[Question ID = 34565][Question Description = M.Sc.MATM_Q_059]

1. S is a finite dimensional vector space over $\mathbb{R}$.
[Option ID = 209169]
2. S is a finite dimensional vector space over $\mathbb{R}$.
[Option ID = 209170]
3. S is not a subspace of V .
[Option ID = 209171]
4. S is a finite dimensional vector space over $\mathbb{C}$.
[Option ID = 209172]

60) Let $U = \{(a, b, c, d) \in \mathbb{R}^4 \mid a - b + c = 0\}$ and $V = \{(a, b, c, d) \in \mathbb{R}^4 \mid a = d, b = -c\}$. Then, the dimension of $U \cap V$ over $\mathbb{R}$ is [Question ID = 34566][Question Description = M.Sc.MATM_Q_060]

1. 1 [Option ID = 209173]
2. 2 [Option ID = 209174]
3. 3 [Option ID = 209175]

4. 0 [Option ID = 209176]

- 61) The co-ordinates of $A=(2,3,4,-7) \in \mathbb{R}^4$ with respect to the basis $\{(1,1,1,1), (1,-1,1,0), (1,-1,0,0), (1,0,0,0)\}$ is given by

[Question ID = 34567][Question Description = M.Sc.MATM_Q_061]

1. $(-7, -11, -22, 30)$. [Option ID = 209177]
2. $(-7, 11, -21, 30)$. [Option ID = 209178]
3. $(7, 11, -21, 31)$. [Option ID = 209179]
4. $(7, -11, -21, 31)$. [Option ID = 209180]

- 62) Let $M_n(\mathbb{C})$ denote the set of all $n \times n$ matrices over $\mathbb{C}$. Let $W = \{A \in M_n(\mathbb{C}) \mid \text{Tr}(A) = 0\}$, where $\text{Tr}(A)$ denote the trace of A . Then, the dimension of W over $\mathbb{R}$ is

[Question ID = 34568][Question Description = M.Sc.MATM_Q_062]

1. n^2 .
[Option ID = 209181]
2. $2n^2-1$.
[Option ID = 209182]
3. n^2-1 .
[Option ID = 209183]
4. $2n^2-2$.
[Option ID = 209184]

- 63) The group $G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \mid a \in \mathbb{R} \right\}$ is isomorphic to

[Question ID = 34569][Question Description = M.Sc.MATM_Q_063]

1. $(\mathbb{C}^*, \times)$.
[Option ID = 209185]
2. $(\mathbb{R}^*, \times)$.
[Option ID = 209186]
3. $(\mathbb{R}, +)$.
[Option ID = 209187]
4. $(\mathbb{C}, +)$.
[Option ID = 209188]

- 64) Let $u=(1,1,0)$, $v=(1,3,2)$, $w=(4,9,5)$ be vectors of $\mathbb{R}^3$. Then, which of the following statements is correct?

[Question ID = 34570][Question Description = M.Sc.MATM_Q_064]

1. The vectors u,v are linearly dependent. [Option ID = 209189]
2. The vectors u,v,w are linearly dependent. [Option ID = 209190]
3. The vectors v,w span $\mathbb{R}^3$.
[Option ID = 209191]
4. The vectors u,v,w are linearly independent. [Option ID = 209192]

- 65) Let S_n denote the symmetric group of permutations on n symbols. Then the least n such that S_n has an abelian subgroup of order 30 is [Question ID = 34571][Question Description = M.Sc.MATM_Q_065]

1. 7 [Option ID = 209193]
2. 9 [Option ID = 209194]
3. 10 [Option ID = 209195]
4. 11 [Option ID = 209196]

- 66) For which of the following group G , G has a subgroup of order d , whenever d divides $O(G)$? [Question ID = 34572][Question Description = M.Sc.MATM_Q_066]

1. A_4 [Option ID = 209197]
2. S_4 [Option ID = 209198]
3. A_5 [Option ID = 209199]
4. A_6 [Option ID = 209200]

67) Which of the following statements are correct?

A. A finite group cannot be written as union of two of its proper subgroups.

B. There is a finite group which can be written as union of three of its proper subgroups. [Question ID = 34573][Question Description = M.Sc.MATM_Q_067]

1. Only A [Option ID = 209201]
2. Only B [Option ID = 209202]
3. None of A and B [Option ID = 209203]
4. Both A and B [Option ID = 209204]

68) Let G be a non-abelian group of order pq , where p and q are distinct primes. Then the number of normal subgroups of G are [Question ID = 34574][Question Description = M.Sc.MATM_Q_068]

1. 1 [Option ID = 209205]
2. $\max\{p, q\}$ [Option ID = 209206]
3. $\min\{p, q\}$ [Option ID = 209207]
4. 2 [Option ID = 209208]

69) Let G be a group of order 45. Then which of the following option is not necessarily true? [Question ID = 34575][Question Description = M.Sc.MATM_Q_069]

1. G has an element of order 9 [Option ID = 209209]
2. G has a subgroup of order 9 [Option ID = 209210]
3. G has a normal subgroup of order 9 [Option ID = 209211]
4. G has a normal subgroup of order 5 [Option ID = 209212]

70) In which of the following group every element is of finite order but group is of infinite order?

[Question ID = 34576][Question Description = M.Sc.MATM_Q_070]

1. The group of non-singular 2×2 matrices with integer entries

[Option ID = 209213]

2. $(\mathbb{Q}/\mathbb{Z}, +)$

[Option ID = 209214]

3. $(\mathbb{Q}, +)$

[Option ID = 209215]

4. $\{z \in \mathbb{C} \mid |z| = 1\}$ with respect to multiplication

[Option ID = 209216]

71)

The matrix A is given by $\begin{bmatrix} 1 & 4 \\ a & 2 \end{bmatrix}$. Then the eigenvalues of A are real and non-negative if

[Question ID = 34577][Question Description = M.Sc.MATM_Q_071]

1. $-\frac{1}{16} \leq a \leq \frac{1}{16}$

[Option ID = 209217]

2. $-\frac{1}{2} \leq a \leq \frac{1}{2}$

[Option ID = 209218]

3. $\frac{-1}{16} \leq a \leq \frac{1}{2}$

[Option ID = 209219]

4. $-\frac{1}{2} \leq a \leq \frac{1}{16}$

[Option ID = 209220]

72) Let A be a 3×4 real matrix and $AX=b$ be an inconsistent system of equations. Then maximum possible rank of A is

[Question ID = 34578][Question Description = M.Sc.MATM_Q_072]

1. 1

[Option ID = 209221]

2. 2

[Option ID = 209222]

3. 3

[Option ID = 209223]

4. 4

[Option ID = 209224]

73) Let A be a 2×2 matrix with eigenvalues

8 and 4 and corresponding eigenvectors respectively are $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Then

[Question ID = 34579][Question Description = M.Sc.MATM_Q_073]

1. $A = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$

[Option ID = 209225]

2. $A = \begin{bmatrix} 6 & 4 \\ 1 & 6 \end{bmatrix}$

[Option ID = 209226]

3. $A = \begin{bmatrix} 4 & 5 \\ 0 & 8 \end{bmatrix}$

[Option ID = 209227]

4. $A = \begin{bmatrix} 8 & 0 \\ 5 & 4 \end{bmatrix}$

[Option ID = 209228]

74) For what values of λ and μ , the system $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ is inconsistent.

[Question ID = 34580][Question Description = M.Sc.MATM_Q_074]

1. $\lambda = 3$ and $\mu = 10$

[Option ID = 209229]

2. $\lambda \neq 3$ and $\mu \neq 10$

[Option ID = 209230]

3. $\lambda = 3$ and $\mu \neq 10$

[Option ID = 209231]

4. $\lambda \neq 3$ and $\mu = 10$

[Option ID = 209232]

75)

The product of nonzero eigenvalues of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

[Question ID = 34581][Question Description = M.Sc.MATM_Q_075]

1. 3 [Option ID = 209233]
2. 4 [Option ID = 209234]
3. 5 [Option ID = 209235]
4. 6 [Option ID = 209236]

76)

The characteristic polynomial of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$

is $-\lambda^3 + 4\lambda^2 + 5\lambda + 6$ then the values of a, b, c are

[Question ID = 34582][Question Description = M.Sc.MATM_Q_076]

1. $a=4, b=5, c=6$
[Option ID = 209237]
2. $a=6, b=5, c=4$
[Option ID = 209238]
3. $a=-4, b=-5, c=-6$
[Option ID = 209239]
4. $a=-6, b=-5, c=-4$
[Option ID = 209240]

77)

Consider a sequence $\{a_n\}$ defined as $a_n = 20 + (-1)^{n^2+1}/(n^2 + 1)$, when n is odd and $a_n = \log_{10}(20 - (-1)^n/n)$, when n is even. Consider the following assertions:

A. $\limsup a_n = \sup a_n$

B. $\liminf a_n = \inf a_n$

Choose the *correct* answer from the options given below:

[Question ID = 34583][Question Description = M.Sc.MATM_Q_077]

1. Both A and B are true [Option ID = 209241]
2. Only A is true [Option ID = 209242]
3. Only B is true [Option ID = 209243]
4. Neither A nor B is true [Option ID = 209244]

78) Which of the following series is convergent: [Question ID = 34584][Question Description = M.Sc.MATM_Q_078]

1. $\sum_2^\infty \frac{1}{n \log n}$

[Option ID = 209245]

2. $\sum_2^\infty \frac{1}{n(\log n)^2}$

[Option ID = 209246]

3. $\sum_1^\infty \cos n$

[Option ID = 209247]

4. $\sum_1^\infty \cos \frac{1}{n^2}$

[Option ID = 209248]

79)

For $a_n = n^{50} e^{-\sqrt{n}}, n \in \mathbb{N}$, the sequence $\{a_n\}$ is

[Question ID = 34585][Question Description = M.Sc.MATM_Q_079]

1. unbounded [Option ID = 209249]
2. bounded and convergent to a positive real number [Option ID = 209250]
3. bounded but does not converge [Option ID = 209251]

4. bounded and convergent to 0 [Option ID = 209252]

80)

Which of the following statements is not true:

[Question ID = 34586][Question Description = M.Sc.MATM_Q_080]

1. Every bounded sequence of real numbers has a monotone subsequence. [Option ID = 209253]

2. If $\{x_n\}$ is a bounded sequence of real numbers with $\sup_n x_n \neq x_m, \forall m \in \mathbb{N}$, then it has a subsequence converging to $\sup_n x_n$.

[Option ID = 209254]

3. Every unbounded sequence of real numbers has a monotone subsequence. [Option ID = 209255]

4. Every monotonic sequence of real numbers has a convergent subsequence. [Option ID = 209256]

81) Let $\{b_n\}$ be a sequence of real numbers. Consider the following statements:

A. $\lim_{n \rightarrow \infty} (b_{n+2} - b_{n+1}) = 0$

B. $|b_{n+2} - b_{n+1}| \leq \frac{2}{7} |b_{n+1} - b_n|, \forall n \geq 1$

C. $|b_{n+1} - b_{n-1}| < |b_n - b_{n-2}|, \forall n \geq 3$

Which of the above statements imply that $\{b_n\}$ is a Cauchy sequence:

[Question ID = 34587][Question Description = M.Sc.MATM_Q_081]

1. A, B and C [Option ID = 209257]

2. Only B [Option ID = 209258]

3. Only B and C [Option ID = 209259]

4. Only A and C [Option ID = 209260]

82) Let $\{x_n\}$ and $\{y_n\}$ be two sequences defined as

$$x_1 = 2, x_{n+1} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right)$$

$$y_1 = 2, y_{n+1} = y_n + \frac{1}{n+1}$$

for all $n \geq 1$. Then which of the following is true:

[Question ID = 34588][Question Description = M.Sc.MATM_Q_082]

1. $\{x_n\}$ and $\{y_n\}$ both are increasing and convergent

[Option ID = 209261]

2. $\{x_n\}$ and $\{y_n\}$ both are convergent with $\lim x_n < \lim y_n$

[Option ID = 209262]

3. $\{x_n\}$ is monotone and convergent where as $\{y_n\}$ is monotone but not convergent

[Option ID = 209263]

4. $\{x_n\}$ is monotone but not convergent where as $\{y_n\}$ is monotone and convergent

[Option ID = 209264]

83) Let $\{a_n\}$ be a decreasing sequence of positive real numbers such that $\sum a_n$ is convergent.

Consider the following statements:

A. $\lim_{n \rightarrow \infty} n a_n = 0$

B. $\sum_{n=1}^{\infty} 2^n a_{2^n}$ converges

C. $\sum \frac{a_n}{1+a_n}$ converges

D. $\sum \frac{a_n}{a_{n+1}}$ converges

Which of the above statements are correct?

[Question ID = 34589][Question Description = M.Sc.MATM_Q_083]

1. A, B and D only [Option ID = 209265]

2. A, B and C only [Option ID = 209266]

3. B and D only [Option ID = 209267]

4. A and B only [Option ID = 209268]

84) Consider the following properties/results:

A. The Monotone Convergence Theorem

B. Completeness of $\mathbb{R}$

C. Nested Interval Property

Which of the above properties are equivalent to each other?

[Question ID = 34590][Question Description = M.Sc.MATM_Q_084]

1. A and B only [Option ID = 209269]
2. A and C only [Option ID = 209270]
3. B and C only [Option ID = 209271]
4. All of A, B and C [Option ID = 209272]

85) Let $\sum a_n$ be a conditionally convergent series of real numbers. statements is not true:

[Question ID = 34591][Question Description = M.Sc.MATM_Q_085]

1. The set $\{n \in \mathbb{N} : a_n > 0\}$ is infinite

[Option ID = 209273]

2. The set $\{n \in \mathbb{N} : a_n < 0\}$ is infinite

[Option ID = 209274]

3. if $b_n = \sum_{k=1}^{10} k a_n^k$, then $\lim_{n \rightarrow \infty} b_n = 0$

[Option ID = 209275]

4. Any rearrangement of $\sum a_n$ is convergent and to the same value

[Option ID = 209276]

86) For $a_n = \frac{(-1)^n}{\sqrt{n}} \left(1 + \frac{1}{n^2}\right)$, which of the following is true:

[Question ID = 34592][Question Description = M.Sc.MATM_Q_086]

1. $\sum a_n$ is conditionally convergent.

[Option ID = 209277]

2. $\sum a_n$ is absolutely convergent.

[Option ID = 209278]

3. The sequence $\{a_n\}$ is bounded but $\sum a_n$ is not convergent.

[Option ID = 209279]

4. The sequence $\{a_n\}$ is unbounded and $\sum a_n$ is divergent.

[Option ID = 209280]

87) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be functions where $f(x) = e^{2x}$ and g is a polynomial of degree n ; $n \geq 2$. Consider the following statements:

A. Cardinality of $\{x \in \mathbb{R} : f \circ g(x) = 1\} \leq n$.

B. If $g(0) = 1, g(1) = 5$, then there exists $x \in]0, 1[$ such that $f \circ g(x) = e^9$.

C. The set $\{x \in \mathbb{R} : f \circ g(x) = 1\}$ is non-empty.

Which of the above statements is/are necessarily true:

[Question ID = 34593][Question Description = M.Sc.MATM_Q_087]

1. A and B only [Option ID = 209281]
2. A and C only [Option ID = 209282]
3. Only A [Option ID = 209283]
4. B and C only [Option ID = 209284]

88)

Let $\{a_n\}$ and $\{b_n\}$ be bounded sequences of real numbers. Which of the following is not true:

[Question ID = 34594][Question Description = M.Sc.MATM_Q_088]

1. $\limsup(a_n + b_n) \leq \limsup(a_n) + \limsup(b_n)$.

[Option ID = 209285]

2. $\liminf(a_n + b_n) \geq \liminf(a_n) + \liminf(b_n)$.

[Option ID = 209286]

3. $\limsup(a_n b_n) = b \limsup(a_n)$ if $\lim_{n \rightarrow \infty} b_n = b$.

[Option ID = 209287]

4. $\liminf(a_n b_n) = \liminf(a_n b)$ if $\lim_{n \rightarrow \infty} b_n = b$.

[Option ID = 209288]

89)

Consider the group $GL_4(\mathbb{R})$ of 4×4 invertible matrices with real entries.

Let H_1 and H_2 be the subgroups of $GL_4(\mathbb{R})$ consisting of diagonal and upper triangular matrices respectively. Choose the correct option.

[Question ID = 34595][Question Description = M.Sc.MATM_Q_089]

1. H_1 is a normal subgroup but H_2 is not a normal subgroup.

[Option ID = 209289]

2. H_2 is a normal subgroup but H_1 is not a normal subgroup.

[Option ID = 209290]

3. Both H_1 and H_2 are normal subgroups.

[Option ID = 209291]

4. Neither H_1 nor H_2 is a normal subgroup.

[Option ID = 209292]

- 90) For a group G , an automorphism of G means a bijective homomorphism from G to G . A subgroup H of G is said to be characteristic if H is mapped to itself under every automorphism of G . Consider the following statements:

- (a) Characteristic subgroups are normal subgroups.
(b) Normal subgroups are characteristic subgroups.

Choose the correct option.

[Question ID = 34596][Question Description = M.Sc.MATM_Q_090]

1. Statement (a) is true, statement (b) is false. [Option ID = 209293]
2. Statement (b) is true, statement (a) is false. [Option ID = 209294]
3. Both statements (a) and (b) are true. [Option ID = 209295]
4. Both statements (a) and (b) are false. [Option ID = 209296]

- 91) Which one of the following is a finitely generated infinite group?[Question ID = 34597][Question Description = M.Sc.MATM_Q_091]

1. $\mathbb{Z}[X]$ with addition of polynomials.

[Option ID = 209297]

2. $\mathbb{Q}$ with usual addition.

[Option ID = 209298]

3. $SL_2(\mathbb{Z})$ with matrix multiplication.

[Option ID = 209299]

4. $\mathbb{R}[X]$ with addition of polynomials.

[Option ID = 209300]

- 92) Consider the following statements:

- (a) Let p be an odd prime number and a be an integer such that $p \nmid a$ and there exists an integer b such that $b^2 \equiv a \pmod{p}$. Then $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.
(b) If for an integer $n > 1$, $(n-1)! \equiv -1 \pmod{n}$, then n is a prime.
Choose the correct option.

[Question ID = 34598][Question Description = M.Sc.MATM_Q_092]

1. Statement (a) is true, statement (b) is false. [Option ID = 209301]
2. Statement (b) is true, statement (a) is false. [Option ID = 209302]
3. Both statements (a) and (b) are true. [Option ID = 209303]
4. Both statements (a) and (b) are false. [Option ID = 209304]

- 93) Consider the following statements:

- (a) For a cyclic group G and a subgroup H , both H and G/H are cyclic.
(b) For a group G and a subgroup H , if both H and G/H are cyclic, then G is a cyclic group.
Choose the correct option.

[Question ID = 34599][Question Description = M.Sc.MATM_Q_093]

1. Statement (a) is true, statement (b) is false. [Option ID = 209305]
2. Statement (b) is true, statement (a) is false. [Option ID = 209306]
3. Both statements (a) and (b) are true. [Option ID = 209307]
4. Both statements (a) and (b) are false. [Option ID = 209308]

- 94) In a ring $(R, +, \cdot)$, an element a is called nilpotent if there exists a positive integer n such that $a^n = \underbrace{a \cdot a \cdots a}_{n \text{ times}} = 0$. Number of nilpotent elements in the ring $\mathbb{Z}/2020\mathbb{Z}, \mathbb{Z}/4040\mathbb{Z}$ are

[Question ID = 34600][Question Description = M.Sc.MATM_Q_094]

1. 2 and 3 respectively. [Option ID = 209309]
2. 3 and 4 respectively. [Option ID = 209310]
3. 2 and 4 respectively. [Option ID = 209311]
4. 1 and 3 respectively. [Option ID = 209312]

- 95) Let T be a linear operator on a finite dimensional vector space V such that $\text{Rank}(T) = \text{Rank}(T^2)$. Consider the following statements:
(a) $\text{Ker}(T) = \text{Ker}(T^2)$
(b) $\text{Ker}(T) \cap \text{Im}(T) = \{0\}$
Choose the correct option.

[Question ID = 34601][Question Description = M.Sc.MATM_Q_095]

1. Statement (a) is true, statement (b) is false. [Option ID = 209313]
2. Statement (b) is true, statement (a) is false. [Option ID = 209314]
3. Both statements (a) and (b) are true. [Option ID = 209315]
4. Both statements (a) and (b) are false. [Option ID = 209316]

- 96) Let T be a linear operator on $\mathbb{R}^4$. For a polynomial $f(X) = a_0 + a_1X + \cdots + a_nX^n \in \mathbb{R}[X]$, let $f(T)$ denotes the operator $a_0I + a_1T + \cdots + a_nT^n$, where I denotes the identity operator on $\mathbb{R}^4$. Consider the following statements:
(a) $f(T)$ can be written as a unique $\mathbb{R}$ -linear combination of I, T, T^2, T^3, T^4 .
(b) $f(T)$ can be written as an $\mathbb{R}$ -linear combination of I, T, T^2, T^3 .
Choose the correct option.

[Question ID = 34602][Question Description = M.Sc.MATM_Q_096]

1. Statement (a) is true, statement (b) is false. [Option ID = 209317]
2. Statement (b) is true, statement (a) is false. [Option ID = 209318]
3. Both statements (a) and (b) are true. [Option ID = 209319]
4. Both statements (a) and (b) are false. [Option ID = 209320]

- 97) Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear operator defined by $T(x_1, x_2, x_3, x_4) = (2x_1 + 9x_3 + 9x_4, 2x_1 + x_2, 2x_1 + x_2 + x_4)$. Choose the correct option.

[Question ID = 34603][Question Description = M.Sc.MATM_Q_097]

1. T is injective but not surjective. [Option ID = 209321]
2. T is surjective but not injective. [Option ID = 209322]
3. T is both injective and surjective. [Option ID = 209323]
4. T is neither injective nor surjective. [Option ID = 209324]

- 98) Let V be the subspace of $\mathbb{R}^3$ generated by $(1, 3, 5)$ and $(2, 3, 4)$. Which vectors among $(3, 4, 5), (1, 2, 4), (2, 4, 6)$ belong to V ?

[Question ID = 34604][Question Description = M.Sc.MATM_Q_098]

1. All of them. [Option ID = 209325]
2. Only $(3, 4, 5), (2, 4, 6)$.

[Option ID = 209326]

3. Only $(3, 4, 5)$.

[Option ID = 209327]

4. Only $(2, 4, 6)$.

[Option ID = 209328]

- 99) Let V be a vector space over $\mathbb{Q}$. Let $T : V \rightarrow V$ be a map such that $T(v_1 + v_2) = T(v_1) + T(v_2)$ for all $v_1, v_2 \in V$. Consider the following statements:

(a) $T(0) = 0$.

(b) $T(-v) = -T(v)$ for all $v \in V$.

(c) T is a linear transformation.

Which among the above statements is/are true?

[Question ID = 34605][Question Description = M.Sc.MATM_Q_099]

1. Only (a) is true. [Option ID = 209329]
2. Only (b) is true. [Option ID = 209330]
3. Only (a) and (b) are true. [Option ID = 209331]
4. All of them are true. [Option ID = 209332]

100)

Let V be a vector space over $\mathbb{Z}/3\mathbb{Z}$ of dimension 3? How many 1 dimensional subspaces does V have?

[Question ID = 34606][Question Description = M.Sc.MATM_Q_100]

1. 26 [Option ID = 209333]
2. 13 [Option ID = 209334]
3. 3 [Option ID = 209335]
4. 9 [Option ID = 209336]